STAT 592: HOMEWORK 1

PROF. HARRY CRANE

1. Homework 1

Problem 1.1. *Write a complete proof for Exercises 1, 2 and 3 in Section 1 of the notes.*

Problem 1.2. *Let P be a set-valued function on a space $(\Omega, \mathcal{F})$ satisfying*

(P1) $P(\Omega) = 1$ *and*
(P2) $P(A) \geq 0$ *for all* $A \in \mathcal{F}$.

Prove the following. P satisfies

(FAP3) $P(A \cup B) = P(A) + P(B)$ *for all* $A, B \in \mathcal{F}$ *such that* $A \cap B = \emptyset$ *and*
(CA) $A_n \uparrow A$ *implies* $P(A_n) \uparrow P(A)$ *for all* $A_n \in \mathcal{F}$ *and* $A = \bigcup_{n\geq 1} A_n$

if and only if

(P3) $P\left(\bigcup_{n\geq 1} A_n\right) = \sum_{n\geq 1} P(A_n)$ *for all* $A_1, A_2, \ldots \in \mathcal{F}$ *that are mutually disjoint.*

Problem 1.3. *Let F be a distribution function such that $0 < F(x) < 1$ for all $x \in \mathbb{R}$ and F^- is its left continuous inverse. Show that*

$$F^-(F(F^-(u))) = F^-(u) \quad \text{for all } u \in (0,1) \quad \text{and}$$
$$F(F^-(F(x))) = F(x) \quad \text{for all } x \in \mathbb{R}.$$

Problem 1.4. *Let X be a random variable with distribution function F and left continuous inverse F^-.*

(i) *Show that for $x \in \mathbb{R}$ and $0 < u < 1$,*

$$u \geq F(x-) \quad \text{if and only if} \quad F^-(u+) \geq x.$$

(ii) *Use part (i) to show that for $0 < u < 1$,*

$$F^-(u+) = \sup\{x : u \geq F(x-)\}.$$

Problem 1.5. *Let $I \subseteq \mathbb{R}$ be a subinterval of the real line. Let f be a nondecreasing mapping $B \to \mathbb{R}$. (For example, f can be a distribution function defined on B, but it need not be.) Put*

$$D_f := \{x \in B : f \text{ is discontinuous at } x\} \quad \text{and}$$
$$C_f := \{x \in B : f \text{ is continuous at } x\}.$$

Prove the following:

(i) *D_f is at most countable.*
(ii) *C_f is dense in B.*