STAT 592: HOMEWORK 6

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1. Homework 6

Problem 1.1. *Let X be a random variable with sample space $\Omega = \{0, 1, \ldots\}$ the set of nonnegative integers. Let $P(t) = E(t^X)$ be its* probability generating function.

1. *Give a motivation for the name of the function P. What is the relationship between the probability generating function P and the moment generating function of X.*
2. *Calculate the probability generating function for the Poisson distribution.*
3. *Is the set where P is defined an interval?*
4. *Let F be the distribution function of X. Define, for $|t| < 1$,*
$$P^*(t) = \sum_{k=0}^{\infty} F(k)t^k.$$
Calculate P^* as a function of P.
5. *Let $X_1, \ldots, X_n$ be i.i.d. with the same distribution as X. Calculate the probability generating function of $X_1 + \cdots + X_n$ as a function of P.*

Problem 1.2. *Calculate the moment generating function for $|X|$, where X is a standard normal random variable, and use it to derive the mean and variance of $|X|$.*

Problem 1.3. *Let $\phi_1, \ldots, \phi_n$ denote characteristic functions for distributions on the real line. Let $a_1, \ldots, a_n$ denote nonnegative constants such that $a_1 + \cdots + a_n = 1$. Show that $\sum_{i=1}^n a_i\phi_i$ is also a characteristic function for some random variable.*

Problem 1.4. *Let $X_{\alpha,\beta}$ have the Beta distribution with parameter α, β. Show that for fixed α and $\beta \to \infty$, $\beta X_{\alpha,\beta}$ converges strongly to X distributed as Gamma with parameter $(\alpha, 1)$.*