Lecture 1

I. Introduction:

A. Probability notions:
   1. Definition: Probability telling us how likely we are to see various outcomes of experiments or studies.

2. Examples: For instance, we might ask
   a. how often will three heads arise in three tosses of a coin? Ask $P(s \in A)$ where $A$ contains all $s$ representing three heads in three tosses of a coin.
   b. how often will a patient’s cancer go into remission after a certain type of chemotherapy? Ask $P(s \in A)$ where $A$ contains all $s$ representing zero tumors after chemotherapy.
   c. how often will a certain company make investments of a certain level and dividends of a certain level? Ask $P(s \in A)$ here $A$ contains all $s$ representing conditions under which that company makes the above-mentioned decisions.

B. Statistical Notions: Statistics is the study of how we can infer about $P$ based on events observed.

1. Typical objectives: testing and estimation.
Lecture 1

a. Ask yes–no questions about $P$: Hypothesis testing.

b. Pick a closest or collection of closest elements from a collection $\mathcal{P}$ of plausible alternative distns: Estimation.

c. Make a decision whose benefit depends on which probability distn is operational: Decision Theory.

2. Examples:

a. Is it plausible that a coin giving heads and tails with equal frequency gave a total # of heads that we observe?

b. Is it plausible that a model of economic decision making in which dividends and investment are unrelated gave rise to an apparent relation between the two observed in the data?

c. Is it plausible that two treatments of equal utility gave rise to a difference in outcomes observed in a medical trial?

II. Probability Theory.

A. Definns:

1. $sample \ space \ S$

   a. Definí: set of outcomes (or $sample \ points$) from an experiment.
b. Examples:

i. Sequence of multiple coin tosses: get things like 
\[ s = HTHHTHT \in S. \]

ii. A medical experiment tests the effectiveness of various treatments. Get results \( s = \) description of the individuals’ health before and after treatment.

iii. An observation on the economy. Get results \( s = \) description of all decisions individuals made on how to spend time, money, and other resources. probability measure \( P \)

2. Events are collections of elements in \( S \).

3. A probability measure \( P(\cdot) \) tells how likely various events are.

B. Examples of how probabilities are represented:

1. Sample space consists of items that can be put in a possibly infinite list:

   a. Examples:

      i. Cards in a deck: 52 items

   b. Each element has a probability value attached.

      i. Calculate probabilities of non-singleton sets by adding the associated probabilities.
ii. Yields a legitimate distribution as long as all singleton probabilities are non-negative and sum to 1.

2. Sample space consists of a region in $\mathbb{R}^k$.
   a. Example: Continuous biological or economic measurement.
   b. Represent probabilities by integrals of a function over event sets: get a legitimate probability if function is non-negative and integrates to 1 over all of $S$.

C. Intuitive requirements for probabilities

1. Something always happens, and denote certainty by a probability of 1: $P(S) = 1$.

2. Require probabilities to be non-negative: $P(A) \geq 0$ if $A \subset S$.

3. Whenever two events cannot happen simultaneously the probabilities of either happening is the sum of the individuals; $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \emptyset$.
   a. Examples:
      i. The first two coin flips are heads vs. the first coin flip is a tail,
      ii. IBM raises dividends vs. IBM cuts dividends,
      iii. A particular patient is cured vs. the same patient stays the same.
D. Consequences of these requirements:

1. \( P(\emptyset) = 0 \), as can be seen by letting \( A_i = \emptyset \) for \( i = 1, \ldots \), and noting that \( P(\emptyset) = \sum_{i=1}^{\infty} P(\emptyset) \).

2. If \( A^c = \{ s \in S | s \notin A \} \) then \( P(A^c) = 1 - P(A) \), since \( P(A) + P(A^c) = P(S) = 1 \).

3. \( P(A) \leq 1 \), since otherwise \( P(A^c) < 0 \).

4. If \( A \subset B \) then \( P(A) \leq P(B) \), since \( P(B) = P(A^c \cap B) + P(A \cap B) = P(A^c \cap B) + P(A) \).