d. Relation to p.d.f. for continuous distributions:

i. \( F_X(x) = \int_{-\infty}^{x} f_X(y) \, dy \).

ii. If \( f_X \) is continuous at \( x_1 \), and if \( x_2 \) is close to \( x_1 \), and 
\( x_2 > x_1 \), then 
\[ F_X(x_2) - F_X(x_1) = P(x_1 < X \leq x_2) = \int_{x_1}^{x_2} f_X(y) \, dy \approx (x_2 - x_1) f_X(x_1) \]

iii. Hence 
\( (F_X(x_2) - F_X(x_1))/(x_2 - x_1) \approx f_X(x_1) \)

iv. Hence 
\( dF_X(x_1)/dx_1 = f_X(x_1) \): Fundamental theorem of calculus.

v. Example: \( f_X(x) = 1 \) for \( x \in (0, 1) \), and equal 0 elsewhere.

- Then
\[ F_X(x) = \begin{cases} 
0 & \text{if } x \leq 0 \\
 x & \text{if } x \in (0, 1) \\
1 & \text{if } x \geq 1 
\end{cases} \]

- Then
\[ F_X'(x) = \begin{cases} 
0 & \text{if } x < 0 \\
1 & \text{if } 1 \in (0, 1) \\
0 & \text{if } x > 1 \\
\text{undefined} & \text{if } x \in \{0, 1\} 
\end{cases} \]: 2.3

M. Describing Distributions:

1. Typical Values

a. The expectation, mean, or average value.
i. Define:

- for continuous dist\(\)s as \(\int x f_X(x) \, dx\)

  - Example: Exponential distribution with \(f_X(x) = \exp(-x)\) for \(x \in [0, \infty)\).

    Integration by parts shows expectation is 1.

- for discrete dist\(\)s as \(\sum x P_X(x)\).

  - Example: Count from one die: \(E[=]1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{21}{6} = 3.5\): Note that this is not a potential data value.

  - Example: Count from two dice:

    | \(x\) | \(P_X(x)\) | \(x P_X(x)\) |
    |-----|----------|-------------|
    | 2   | 1/36     | 1/18        |
    | 3   | 2/36     | 3/18        |
    | 4   | 3/36     | 6/18        |
    | 5   | 4/36     | 10/18       |
    | 6   | 5/36     | 15/18       |
    | 7   | 6/36     | 21/18       |
    | 8   | 5/36     | 20/18       |
    | 9   | 4/36     | 18/18       |
    | 10  | 3/36     | 15/18       |
    | 11  | 2/36     | 11/18       |
    | 12  | 1/36     | 6/18        |

    Sum of last column is \(\frac{128}{18} = 7\).

  - Note that expectation is point of symmetry, if distribution is...
symmetric.

- Note that expectation of sum is sum of expectation: always holds.