III. Standard discrete distributions:

A. Bernoulli random variable/distribution:
1. Outcome of one 1 or 0 experiment with known probability \(p\) of 1.
2. \(E[X] = 1 \times p + 0 \times (1 - p) = p\)
3. \(X^2 = X \rightarrow E[X^2] = E[X] = p\)
4. \(\text{Var}[X] = p - p^2 = p(1-p)\).

B. Binomial \(\text{Bin}(n, p)\)
1. Definition:
   a. sum of \(n\) independent Bernoulli \((p)\) trials.
   b. count of successes in \(n\) independent Bernoulli \((p)\) trials.
2. To get pmf
   a. Probability of \(x\) successes is the sum of the probabilities of all strings of outcomes giving the same number of successes.
   b. Each of these strings has probability \(p^x(1-p)^{n-x}\).
   c. There are \(\binom{n}{x}\) such strings
   d. pmf \(p_X(x) = \binom{n}{x}p^x(1-p)^{n-x}\).
3. cdf: \(F_x(x) = \sum_{v=0}^{x} \binom{n}{v}p^v(1-p)^{n-v};\) no elementary simplification is available.
   a. Evaluate using standard software
      i. Ex., on eden, at the command line, type \(R\) to enter \(R\) program; then you can get \(P(X \leq x)\) for \(X \sim \text{Bin}(n, p)\) by typing \( \text{pbinom}(x, n, p). \)
4. Expectation: \(E[X] = np\), since \(X\) is the sum of \(n\) random variables each with expectation \(p\).

5. Variance: \(\text{Var}[X] = np(1-p)\), since \(X\) is the sum of \(n\) independent random variables each with expectation \(p(1-p)\).

C. Geometric
1. Waiting time to first success in a stream of independent Bernoulli \((p)\) trials.
2. To get pmf,
   a. the only sequence of Bernoulli trials for a given value of \(x\) has \(x-1\) failures followed by one success,
   b. \(p_X(x) = p(1-p)^{x-1}\).
3. cdf \(P(X \leq x) = 1 - P(X > x) = 1 - P(\text{all of first } x \text{ trials failures}) = 1 - (1-p)^x\).

D. Negative binomial
1. Waiting time to first \(r\) successes in a stream of independent Bernoulli \((p)\) trials.
2. To get pmf,
   a. a sequence of Bernoulli trials for a given value of \(x\) has a mixture of \(x-r\) failures and \(r-1\) successes,
followed by one success,
b. \( p_X(x) = \binom{x-r}{r-1}p^r(1-p)^{x-r} \)

3. Expectation: time until \( r \) successes is sum of \( r \) waits until next success;
a. each of these waits is geometric and independent
   (independence we don’t need until we do variance).
   \( E[X] = r/p \)

4. Variance \( \text{Var}[X] = n(1-p)/p^2 \).