b. \[ A = 2^{1/2} \int_0^\infty \exp(-w)w^{-1/2} \, dw \text{ for } w = z^2/2, \\
\quad z = \sqrt{2w}, \quad dz = 2^{-1/2}w^{-1/2} \, dw. \]

2. Clearly if \( X \sim \text{N}(\mu, \sigma^2) \) then the median of \( X \) is \( \mu \), by symmetry.

3. \( \mathbb{E}[X] = \mu \), again by symmetry, if expectation exists
   a. That is, if \( \int_{-\infty}^{\infty} x^2 \sigma^{-1}(2\pi)^{-1/2} \exp(-(x-\mu)^2/(2\sigma^2)) \, dx < \infty \)
   b. Integral is finite, by comparing with integral with \(-| \cdot | \) replacing \(-(\cdot)^2\) in exponent.

4. \[ \text{Var}[X] = \int_{-\infty}^{\infty} (x - \mu)^2 \exp(-(x-\mu)^2/(2\sigma^2)) \, dx \\
\quad = \sigma^2 \int_{-\infty}^{\infty} z^2(2\pi)^{-1/2} \exp(-z^2/2) \, dz \]
and use integration by parts, with \( u = z \) and \( v = \exp(-z^2/2) \) to show that the integral is \( 1 \).

5. Note that many of these explorations began by changing variables to the case with \( \mu = 0 \), and \( \sigma = 1 \).
   a. This case is known as standard normal.
   b. If \( X \sim \text{N}(\mu, \sigma^2) \) and \( Y = aX + b \) for \( a \neq 0 \) then \( Y \sim \text{N}(a\mu + b, a^2\sigma^2) \)
      i. Here \( Y = g(X) \) for \( g(x) = ax + b \), and \( g^{-1}(y) = (y - b)/a \).
      ii. Use rule \( f_Y(y) = f_X(g^{-1}(y)) \frac{dy}{dx} = \frac{\exp(-((y-b)/a-\mu)^2/(2\sigma^2))}{\sqrt{2\pi}\sigma} \)
   c. If \( X \sim \text{N}(\mu, \sigma^2) \) then \( Y = (X - \mu)/\sigma \sim \text{N}(0, 1) \).
   d. Then \( F_X(x) = P(X \leq x) = P(Z \leq (x-\mu)/\sigma) = \Phi((x-\mu)/\sigma) \)
   e. Denote \( F_Z(z) \) by \( \Phi(z) \), the standard normal cdf.
   f. So \( F_X(x) = \Phi((z - \mu)/\sigma). \)
   g. \( \Phi \) is tabulated in book.

6. Linear combination of two independent standard normal random variables is normal
   a. Suppose \( X \sim \text{N}(0, 1), \; Y \sim \text{N}(0, 1), \; X \perp Y, \; Z = aX + bY \); \( b > 0, \; a \neq 0 \). Then \( Z \sim \text{N}(0, a^2 + b^2) \).
      i. Integrate over region with \( x \in (-\infty, \infty), \; y \in (-\infty, (z - ax)/b) \):
      \[ P(Z \leq z) = \int_{-\infty}^{\infty} \int_{-\infty}^{(z - ax)/b} \frac{\exp(-x^2/2 - y^2/2)}{2\pi} \, dy \, dx \]
      \[ = \int_{-\infty}^{\infty} \int_{-\infty}^{z} \frac{\exp(-x^2/2 - (w - ax)^2/2)}{2\pi} \, dw \, dx \]
      \[ = \int_{-\infty}^{\infty} \int_{-\infty}^{z} \frac{\exp(-u^2/2)}{2\pi} \, b^{-1} \, dw \, dx \]
      \[ = \int_{-\infty}^{\infty} \frac{\exp(-aw^2/2)}{2\pi} \, b^{-1} \, dw \]
      \[ = \int_{-\infty}^{\infty} \frac{\exp(-aw^2/2)}{2\pi} \, b^{-1} \, dw \]

7. Linear combination of any number of independent general normal random variables is normal
   a. Recall that means and variances both add.
   b. So if \( X_1 \) are independent \( \text{N}(\mu_i, \sigma_i^2) \) then \( \bar{X} = \sum_{i=1}^{n} X_i/n \sim \text{N}(\mu, \sigma^2/n). \)