1. Question 3.5.1 (Hayter), page 180. A garage sells three types of tires, type A, type B, and type C. A customer purchases type A with probability 0.23, type B with probability 0.48, and type C with probability 0.29.

a. What is the probability that the next 11 customers purchase four sets of type A, five sets of type B, and two sets of type C.

The probability is

\[
\binom{11}{4, 5, 2}(0.23)^4(0.48)^5(0.29)^2 = \frac{11!}{4!5!2!}(0.23)^4(0.48)^5(0.29)^2.
\]

Simplifying, this equals

\[
11 \times 5 \times 3 \times 7 \times 6(0.23)^4(0.48)^5(0.29)^2 = 0.04156.
\]

In R, do `dmultinom(c(4, 5, 2), prob=c(0.23, 0.48, 0.29))`.

b. What is the probability that fewer than three sets of type A are sold to the next seven customers.

The marginal distribution for Type A is binomial with probability 0.23. Hence

\[
P(\text{Fewer than three sets of A}) = (0.77)^7 + 7(0.77)^6(0.23) + \frac{7 \times 6}{2}(0.77)^5(0.23)^2 = 0.797.
\]

Almost 80% of the time two or fewer customers will buy type A. This might also be done using `pbinom(2, 7, .23)`.

2. Question 3.5.2 (Hayter), page 180. A fair die is rolled 15 times. Calculate the probability that there are:

a. Exactly 3 6s and three 5s.

Consider the die outcomes as 6, with probably 1/6 , 5, with probably 1/6 , and anything else, with probably 2/3 . The the probability of 3 6s, 3 5s, and 9 of anything else, is

\[
\frac{15!}{3!3!9!}(4/6)^9(1/6)^3(1/6)^3.
\]

. Simplifying the fraction gives

\[
\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 6} \times \frac{4^9}{6^9} = 5 \times 14 \times 13 \times 11 \times 10 \times \frac{4^9}{6^{15}} = 0.0558.
\]

You might do this in R via

`dmultinom(c(9, 3, 3), prob=c(4, 1, 1)/6)`.

b. Exactly 3 6s, three 5s, and four 4s.

Consider the die outcomes as 6, with probably 1/6 , 5, with probably 1/6 , 4, with probably 1/6 , and anything else, with probably 1/2 . The the probability of 3 6’s, 3 5’s, 4 4’s, and 5 of anything else, is

\[
\frac{15!}{3!3!4!5!}(3/6)^5(1/6)^4(1/6)^3(1/6)^43^5/6^{15}.
\]
Simplifying the fraction of factorials gives
\[
5 \times 7 \times 13 \times 11 \times 5 \times 9 \times 8 \times 7 \times 3^5 / 6^{15} = 0.0065.
\]
You might do this in R via
\[
dmultinom(c(5,4,3,3),\text{prob}=c(3,1,1,1)/6).
\]

3. Question 1 (DeGroot), page 306. Suppose that two different tests \( A \) and \( B \) are to be given to a student chosen at random from a certain population. Suppose also that the mean score on test \( A \) is 85 and the standard deviation is 10; that the mean score on test B is 90 and the standard deviation is 16; that the scores on the two tests have a bivariate normal distribution; and that the correlation of the two scores is 0.8. If a student’s score on test \( A \) is 80, what is the probability that his score on test \( B \) will be higher than 90 ?

Class notes show that the distribution of \( B \) conditional on \( A = a \) is normal with expectation \( \mu_B + \sigma_B \rho(a - \mu_A)/\sigma_A = 90 + 16 \times .8(80 - 85)/10 = 83.6 \) and variance \( (1 - \rho^2)\sigma_B^2 = (1 - .64) \times 100 = 36 \). Then \( P(B \geq 90|A = 80) = 1 - \Phi((90 - 83.6)/6) = 1 - \Phi(1.067) = 0.143 \); one could do this with \( 1-\text{pnorm}(90,83.6,6) \).

4. Question 3 (DeGroot), page 306. Consider again the two tests \( A \) and \( B \) described in Exercise 1. If a student is chosen at random, what is the probability that his score on test \( A \) will be higher than his score on test \( B \) ? (Hint: Suppose that the mean score on test \( A \) is 85 and the standard deviation is 10; that the mean score on test B is 90 and the standard deviation is 16; that the scores on the two tests have a bivariate normal distribution; and that the correlation of the two scores is 0.8.)

Let \( D = B - A \). We want \( P(D < 0) \). Note that \( V(B - A) = 1^2 V(B) + (-1)^2 V(A) + 2 \times 1 \times (-1) \times \text{Cov}(A,B) \). Then
\[
D \sim N(90 - 85, 16^2 + 10^2 - 2 \times 10 \times 16 \times .8) = N(5,100 + 256 - 256) = N(5,100).
\]
Hence \( P(D < 0) = P((D - 5)/10 < -5/10) \), with \((D - 5)/10 \) standard normal. This probability is 0.3085, as can be seen from a standard normal table, or \( \text{pnorm}(-.5) \), or \( \text{pnorm}(0,5,10) \).