1. Question 1 (DeGroot), page 285f. Let $X$ denote the total number of successes in 15 Bernoulli trials, with probability of success $p = 0.3$ on each trial.

a. Determine approximately the value of $P(X = 4)$ by using the central limit theorem with the correction for continuity.

Let $Z = \frac{(X - 0.3 \times 15)}{\sqrt{15 \times 0.3 \times 0.7}}$. Approximate the probability $P(X = 4)$ as

$$P(X = 4) = P(X \leq 4.5) - P(X \leq 3.5).$$

Converting to standard normal variates, this is

$$P\left(Z \leq \frac{4.5 - 4.5}{1.775}\right) - P\left(Z \leq \frac{3.5 - 4.5}{1.775}\right).$$

From the table or R, this is $0.5 - \Phi(-1/1.775) = 0.5 - 0.287 = 0.213$.

b. Compare the answer obtained in part (a) with the exact value of the probability.

Compare this with the exact value of 0.219.

2. Question 5.3.1 (Hayter), page 248. Calculate the following probabilities both exactly and by using a normal approximation:

a. $P(X \geq 8)$ for $X \sim \text{Bin}(10, 0.7)$.

Let $Z = \frac{(X - 7)}{\sqrt{10 \times 0.7 \times 0.3}} = (X - 7)/1.449$. Then

$$P(X \geq 8) = P\left(Z \geq \frac{7.5 - 7}{1.449}\right) = 1 - P(Z \leq 0.345) = 0.365.$$  

Compare with $1 - \text{pbinom}(7,10,0.7)=0.383$.

b. $P(2 \leq X \leq 7)$ for $X \sim \text{Bin}(15, 0.3)$.

Let $Z = \frac{(X - 4.5)}{\sqrt{15 \times 0.7 \times 0.3}} = (X - 4.5)/1.775$. Then

$$P(2 \leq X \leq 7) = P\left(\frac{1.5 - 4.5}{1.775} \leq Z \leq \frac{7.5 - 4.5}{1.775}\right).$$

Evaluating the fractions gives

$$P(Z \leq 1.690) - P(Z \leq -1.690) = 0.909.$$  

Compare with $\text{pbinom}(7,15,0.3) - \text{pbinom}(1,15,0.3)=0.915$.

c. $P(X \leq 4)$ for $X \sim \text{Bin}(9, 0.4)$.

Let $Z = \frac{(X - 3.6)}{\sqrt{9 \times 0.4 \times 0.6}} = (X - 3.6)/1.470$. Then

$$P(X \leq 4) = P(Z \leq (4.5 - 3.6)/1.470) = P(Z \leq 0.612) = 0.730.$$  

Compare with $\text{pbinom}(4,9,0.4)=0.733$.

d. $P(8 \leq X \leq 11)$ for $X \sim \text{Bin}(14, 0.6)$.
Let \( Z = (X - 8.4)/\sqrt{14 \times 0.6 \times 0.4} = (X - 8.4)/1.883 \). Then
\[
P(8 \leq X \leq 11) = P\left(\frac{7.5 - 8.4}{1.883} \leq Z \leq \frac{11.5 - 8.4}{1.883}\right).
\]

Evaluating the fractions gives
\[
P(Z \leq 1.646) - P(Z \leq -0.478) = 0.634.
\]

Compare with \( \text{pbinom}(11, 14, 0.6) - \text{pbinom}(7, 14, 0.6) = 0.653 \).

3. Question 2 (DeGroot), page 281. Suppose that the distribution of the number of defects on any given bolt of cloth is a Poisson distribution with mean 5, and that the number of defects on each bolt is counted for a random sample of 125 bolts. Determine the probability that the average number of defects per bolt in the sample will be less than 5.5.

Recall that the variance of the Poisson distribution is the same as the expectation; in this case the common value is 5. Hence the distribution of interest has expectation 5 and standard deviation \( \sqrt{5/125} \). The probability of fewer than 5.5 defects per bolt is
\[
\Phi((5.5 - 5)/\sqrt{5/125}) = 0.9937.
\]
The correction for continuity is \( 1/(2 \times 125) \), and is small enough to be ignored.