Lecture 19 181

4. Moments of Sums of Two Variables

i. Variance of Sum is the sum of variances plus twice the covariance.

\[ \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y) \]

ii. Let \( X \) and \( Y \) be random variables,
\[ \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y) \]

iii. Distributing, \( \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y) \)

iv. Express the square as a product with different indices:
\[ \text{Var}(X + Y) = \sum_{i=1}^{n} (X_i - \mu_i)^2 \]

v. Expand the double sum into the sum over all pairs of \( i \) and \( j \):
\[ \text{Var}(X + Y) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} a_i a_j \text{Cov}(X_i, X_j) \]

vi. Pairs with \( i = j \) give variances.

vii. Hence
\[ \text{Var}(X + Y) = \sum_{i=1}^{n} a_i^2 \text{Var}(X_i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i a_j \text{Cov}(X_i, X_j) \]

8. Variances add for sums of independent random variables.

a. Take \( X_1, \ldots, X_n \) are independent, \( a_i \) constants.

b. Then \( \text{Var}(X_1 + \ldots + X_n) = \sum_{i=1}^{n} a_i^2 \text{Var}(X_i) \)

c. Because covariances are zero.