Lecture 12

- The solution is

\[ \theta = \frac{Q + z_{\alpha/2}/(2m) \pm \sqrt{z_{\alpha/2}^2/m}}{1 + z_{\alpha/2}^2/m} \sqrt{Q(1 - Q) + \frac{1}{4}z_{\alpha/2}^2/m} \]

\[ \approx Q \pm \sqrt{z_{\alpha/2}^2/m} \sqrt{Q(1 - Q)} \]

iv. Rule of Thumb: Harder approximation works if \( mQ > 5 \) and \( (1 - m)Q > 5 \).

v. Coverage for easy interval is bad for extreme \( \theta \)
   - For \( \theta > 0 \) but very small,
   - CI for \( X = 0 \) is only 0
   - \( P[X = 0] = (1 - \theta)^m \)
   - Hence \( P[\theta \in \text{CI}] \leq (1 - \theta)^m \to 0 \).

vi. Let \( Y_1, \ldots, Y_m \) be 0s and 1s
   - Sample variance of \( Y_j \) is \( \left[ \sum_{j=1}^m Y_j^2 - (\sum_{j=1}^m Y_j)^2/m \right]/(m-1) = [X - X^2/m]/(m-1) = \frac{m}{m-1}(X/m)(1 - X/m)/m \)
   - Hence easy version of the CI differs from CI for mean by factor of \( \frac{m}{m-1} \) in SE

\[ \text{F: 11.5} \]

f. Example: Differences between binomial proportions
Lecture 12

True Confidence Levels for Nominal 95% Binomial Intervals

True $1 - \alpha$

Success Probability

3 observations

50 observations

10 observations

100 observations
Lecture 13

i. $X \sim \text{Bin}(\theta, m), \ Y \sim \text{Bin}(\tau, n)$

ii. Want CI for $\tau - \theta$

iii. Use same idea as before: Approximately,

$$\frac{(Y/n - X/m) - (\tau - \theta)}{\sqrt{\theta(1 - \theta)/m + \tau(1 - \tau)/n}} \sim \mathcal{N}(0, 1).$$

iv. No way to solve quadratic part for $\tau - \theta$

v. Use same idea as before: Approximately,

$$\frac{(Y/n - X/m) - (\tau - \theta)}{\sqrt{(X/m)(1 - X/m)/m + (Y/n)(1 - Y/n)/n}} \sim \mathcal{N}(0, 1).$$