Power for mean and median, $n = 9$

Normal sample of size 9
7. Corollary: Most powerful test for two simple hypotheses depends only on sufficient statistic

a. If $T$ is sufficient, then $L(X, \theta) = g(T, \theta)h(X, T)$

b. Likelihood ratio test statistic is

$$\Lambda = \frac{L(X, \theta_A)}{L(X, \theta_0)} = \frac{g(T, \theta_A)h(X, T)}{g(T, \theta_0)h(X, T)} = \frac{g(T, \theta_A)}{g(T, \theta_0)}$$

F: 12.6

D. Testing for composite hypotheses: Formal Definition:

1. To test when l.r.t. depends on alternative:

2. Assume $X \sim f(x; \theta)$ for some $\theta$.

3. Test $\theta \in \omega$ vs $\theta \in \omega'$.

   a. Procedure:

   b. Define $\Lambda = (\max_{\theta \in \omega} L(\theta; X))/(\max_{\theta \in \Omega} L(\theta; X))$.

   c. Reject when small.

   d. For example, in binomial case, when testing $H_0 : \pi = \pi_0$ vs. $H_A : \pi \neq \pi_0$?

      i. Since the 1/ tests against different alternatives take different forms, there won’t be a uniformly most powerful test.

   e. Hypotheses
Lecture 18

i. $H_0 : \theta \in \omega$

ii. $H_A : \theta \in \omega'$

iii. Let $\Omega = \omega \cup \omega'$.

f. Modify the likelihood ratio approach

i. also called the likelihood ratio test

ii. Substitute the $\theta$ value in each hypothesis maximizing $L(\theta)$

• $\hat{\theta} = \text{argmax}_\omega L$

• $\tilde{\theta} = \text{argmax}_\Omega L$

iii. and then taking ratios and inverting.

iv. reject $H_0$ when small.

F: 13.5

4. Example: $X \sim \text{Bin}(m, \pi)$, $H_0 : \pi = \pi_0$, $H_A : \pi \neq \pi_0$,

a. then $\hat{\pi} = \pi_0$ maximizes $L$ over $\omega$,

b. then $\tilde{\pi} = X/m$ maximizes $L$ over $\Omega$,

c. and so

$$\Lambda = \frac{\pi_0^X (1 - \pi_0)^{m-X} \binom{m}{X}}{\tilde{\pi}^X (1 - \tilde{\pi})^{m-X} \binom{m}{X}}$$

$$= \frac{\pi_0^X (1 - \pi_0)^{m-X}}{(X/m)^X (1 - X/m)^{m-X}}$$
d. Equivalent to test based on $\Lambda^{1/m} = \frac{\pi_0^Q (1-\pi_0)^{1-Q}}{Q^Q (1-Q)^{1-Q}}$ for $Q = X/m$
e. As before the test statistic is a function of the sufficient statistic.

f. Unlike before it’s not a simple function, and finding the c.r. will be difficult.