2. Formal statement of problem:
   a. Given data $X_1, \ldots, X_n$ from a model $X_i \sim \text{Bin}(m, \theta)$
   b. wish to test the hypothesis, called the null hypothesis, that $\theta$ takes on a value in some set, against the alternative hypothesis that $\theta$ is in some other set.

   i. Example: Confidence Interval for Log Odds
      1. $X \sim \text{Bin}(m, \theta)$
      2. Get confidence interval for $\psi = \log(\theta) - \log(1 - \theta)$
      3. Let $\hat{\psi} = \log(X/m) - \log(1 - X/m)$
      4. $\log(X/m) - \log(1 - X/m) \approx \log(\theta) - \log(1 - \theta) + (1/\theta + 1/(1 - \theta))(X/m - \theta)$
   v. Hence
   P \left[ \hat{\psi} - \psi \leq y \right] \approx P \left[ (1/\theta + 1/(1 - \theta))(X/m - \theta) \leq y \right] = P \left[ (X/m - \theta) \leq y/(1/\theta + 1/(1 - \theta)) \right] = P \left[ (X/m - \theta)/\sqrt{\theta(1-\theta)/m} \leq y\sqrt{m\theta(1-\theta)} \right] = P\left( z \leq y \right)
   vi. Hence $1 - \alpha$ confidence interval for $\psi$ is $\hat{\psi} \pm \frac{z_{\alpha/2}}{mX/m(1-X/m)}$.
   vii. Size $F: 12.1$

3. What makes a good test? Among test of a fixed size:
   a. Why is the alternative of randomly rejecting the same probability, without regard to data, a bad test?
   b. Want the Type II error rate small, or alternatively, want the power, or probability of correct decision under the alternative.

4. General Construction
   a. Create a test statistic $L$ that gives more evidence against $H_0$ the bigger it is,
   b. Rejecting $H_0$ if the statistic is equal to or larger than a threshold value, called the critical value.
   F: 12.2

g. Type I error rate called size.

5. Decision-Theoretic Approach
   a. Make loss function that depends on choice $a_0$, $a_1$, and truth $\theta_0$, $\theta_1$

6. Example: Binomial Case:
   a. Problem:
      i. $X \sim \text{Bin}(m, \pi)$
      ii. $H_0 : \pi = \pi_0 (= .65)$, $H_A : \pi > \pi_0$.
      iii. Type I error rate $\alpha$
   b. Use as test statistic observed defective proportion $Q$
      i. Find the critical value $c$ to make the test “Reject if $mQ = X \geq c$” have size $\alpha$.
      ii. Equivalently, ask for the value of $C$ such that under $H_0$, $P[Q \geq C] = \alpha$ for $C = c/m$.
iii. Via approximation, need $C$ such that
\[ P[Q \geq C] \approx \alpha. \]

- Since \( (Q - \pi)/\sqrt{\pi(1-\pi)/m} \sim N(0, 1) \),
  \[ P\left[(Q - \pi)/\sqrt{\pi(1-\pi)/m} \geq z_\alpha\right] \approx \alpha, \]
- and \( P\left[Q \geq \pi + z_\alpha \sqrt{\pi(1-\pi)/m}\right] \approx \alpha; \)
- hence \( \pi + z_\alpha \sqrt{\pi(1-\pi)/m} \) is the approximate critical value.