3. Bootstrap estimates of standard error
   a. How have we been calculating test size and power all semester?
   b. Sample B sets of observations from statistical model
      i. under null hypothesis for size
      ii. under alternative hypothesis for power
   c. For each sampled data set, calculate test statistic
   d. Count how often new test statistics exceed critical value.

4. Could do the same thing with variance of estimator \( \hat{\theta} \).
   a. Figure out true value for parameter.
   b. Sample observations from statistical model
   c. For sample \( i \), calculate estimator \( \hat{\theta}_i \).
   d. Calculate variance of these estimators.

5. Problems:
   a. In many cases, we are not prepared to make assumptions about the distribution of the observations.
   b. If we knew the true values of the parameter, we would not have to do statistics.

D. Which order statistic from the bootstrap sample do we use?
1. Most naive approach uses \( \hat{\theta}_{B(i)} \) to represent quantile \( i/N \) of this distribution.
   a. \( \hat{\theta}_{B(1)} \) represents 1/N quantile
   b. \( \hat{\theta}_{B(N)} \) represents 1 quantile
2. Conceptually, the estimation problem aught to be symmetric if we swap the order of bootstrap observations
   a. Naive quantiles are not.

Lecture 11

\[ \beta. \]

10. Then the \( \hat{\theta} \) in the upper bound is \( \hat{\theta}_{B(i+1)} \).
11. Can do other tricky things, like Coefficient of Variation.

F. Bootstrap- \( t \) confidence interval:
1. Setup:
   a. \( \hat{\theta} \) has distribution centered around \( \theta \).
   b. but with dispersion \( \sigma(\theta) \) depending on \( \theta \)
   c. You have an estimate of \( \alpha \) of \( \sigma(\theta) \)
   d. Ratio is called studentized.
2. Resample \( B \) data sets as above.
3. For each resampled data set, calculate
   a. estimate \( \hat{\theta}_{B,i} \)
   b. standard error \( \hat{\sigma}_{B,i} \).
4. Let empirical distribution of \( T_{B,i} = (\hat{\theta}_{B,i} - \hat{\theta})/\hat{\sigma}_{B,i} \) stand in for distribution of \( T = (\hat{\theta} - \theta)/\sigma \)
   a. This distribution doesn’t involve unknown parameters
      i. Such a quantity is called pivotal.
      ii. Typical strategy for inference.
   b. With \( \beta \) in for \( \mu \).
   c. Then \( .95 \approx P \left[ t_L \leq (\hat{\theta} - \theta)/\hat{\sigma} \leq t_U \right] \), for \( t_L \) and \( t_U \)
      the .025 and .975 points of the distribution of \( T \).
   d. Let \( t_L \) and \( t_U \) be percentage points from bootstrap distribution.
5. Often have \( \hat{\theta} = E[X_j], \hat{\sigma} = \bar{X}, \hat{\sigma} = S/\sqrt{n}. \)  

G. Can get scale parameter via log scale.
explanatory variables are more variable than others.

v. Partial Solution:
   • Calculate $h_j = \text{diagonal elements of the matrix } I - X(X^\top X)^{-1}X^\top$.
   • Divide residual by $\sqrt{1 - h_j}$ for $h_j$ the "hat value".

vi. Create new response variable by adding resampled residuals to old fitted values

vii. Fit regression line to new response and old explanatory variable

e. Use same methods as above for inference.

3. Either the method of resampling new observation vectors, or resampling new residuals, works for multiple regression.
   a. Method can be used to get CI for
      i. one or more regression coefficients,
      ii. $R^2$,
      iii. etc.

4. In order to use the bootstrap-t confidence interval method, retain not only the parameter estimates, but the standard errors as well.