d. Conditionality principal: If

i. data arises from random mixture of experiments
   • Here indexed by $d_+$

ii. mixing distribution does not depend on unknown parameter

iii. Then perform inference based on experiment we see

e. $p$-value $2 \times \min(P[d_0 \geq \text{observed}] , P[d_0 \leq \text{observed}])$
   i. $= 2\Phi(-|\text{observed} - \text{observed}| / \sqrt{d_+\pi(1 - \pi)})$.
   ii. To properly account for probability at $\text{observed}$, add $\pm \frac{1}{2}$ to numerator to make absolute value smaller. See Figure 3.

f. Get CI for $\pi$ using

i. Normal approx. $\pi \in d_1/d_+ \pm 1.96 \sqrt{\frac{d_0d_1}{(d_1+d_0)^3}}$
   • Problem if $d_0 = 0$
   • Less obvious problem for small $d_1 + d_0$

ii. Fix problem by working exactly:
   • Lower bound $\pi_L$ satisfies $P_{\pi}[d_0 \geq \text{observed}] = .025$
   • Upper bound $\pi_U$ satisfies $P_{\pi}[d_0 \leq \text{observed}] = .025$
   • Vertical line has probability .95 for any value of parameter
   • Hence horizontal line has same coverage
   • Lower confidence bound is generated by upper quantile and
vice versa

- See Figure 4.
- Can be expressed in terms of $F$ distribution upper tail:
iii. Are intermediate policies between these extremes.

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g. Get CI for $q$ using $q = e_0\pi / [e_1(1 - \pi)]$ evaluated at upper and lower CI of $\pi_1$

i. Works since relationship between $\pi_1$ and $q$ is strictly increasing.
Lecture 4

h. Special Case: one age group
   i. Then population SMR = relative risk for exposure group relative to standard population
   ii. Then ratio of population SMR = relative risk for exposure groups

5. Multiple (K) Exposure Categories
   a. How do exposure groups differ?
      i. Choose one group as baseline
         • Usually the one with no exposure, if there is one
         • Be careful what you lump in here
      ii. Calculate relative risks with respect to this group
   b. Wrong answer:
      i. Calculate hypothesis tests
         • for each pair
         • or against a baseline
      ii. Claim heterogeneity if any of these shows up different
   c. To avoid multiple comparisons, need one test for all groups
   d. Choose a measure of disagreement with null answer
i. Calculate Expected value
   - Expected value in light of all coming from this non–standard cohort
   - Hence don’t expect each rate to be associated expectation
   - Expect each rate to be \( \propto \) associated expectation
   - \( E_k = d_k Q_k / \sum_j Q_j \)

ii. Use as test statistic sum distances from expectation
   - squared
   - weighted by estimated variance
   - \( \sum_k (d_k - E_k)^2 / E_k \)
   - Distribution is that of sum of \( K \) squared \( \mathcal{N}(0, 1) \)
     - Not independent
     - Equivalent to \( K - 1 \) independent \( \mathcal{N}(0, 1)^2 \)
     - Distribution called \( \chi^2 \) on \( K - 1 \) degrees of freedom

e. Why not CI?
   i. CI can give test when we have one parameter to test
   ii. Here we need \( K - 1 \) parameters
   iii. CI becomes confidence region: more complicated.

f. Exact methods?
i. Same test statistic

ii. Distribution in cells is given by sequence of binomials

iii. Hard to calculate

g. When $K = 2$:

i. $d_1 = d_+ - d_0$ and $E_1 = d_+ - E_0$.

ii. $E_1 = d_+ \pi$

iii. $T = (d_0 - E_0)^2/E_0 + (d_1 - E_1)^2/E_1 = (d_0 - E_0)^2[1/E_0 + 1/E_1] = (d_0 - E_0)^2d_+^{-1}[1/\pi + 1/(1-\pi)] = (d_0 - E_0)^2d_+^{-1}/(\pi(1-\pi))$

iv. Hence $\chi^2$ statistic is square of $Z$ statistic

v. Hence inference is the same.