3. General Problem:
   a. Two “preparations”
   b. Continuous measure of outcome
   c. Determine ratio of doses necessary to obtain equivalent outcome: potency
4. Parallel–Line Bioassay
   a. Results of two preparations at various doses are measured
   b. Want to report ratio of preparations needed to get identical results: potency \( \exp(\omega) \)
   c. Model:
      i. Standard preparation
         \[ Y_{0j} = \beta_0 X_0^\beta_1 + \epsilon_{ij} \]
      ii. New preparation
         \[ Y_{1j} = \beta_0(\exp(\omega) X_{1j})^{\beta_1} + \epsilon_{ij} \]
   d. When \( \beta_1 = 0 \) you have a problem
      i. No change in mean value as \( X_{ij} \) changes
   e. Combined model, on log scale:
      \[ \log(Y_{ij}) = \log(\beta_0) + \beta_1 \log(X_{1j}) + \beta_2 i + \epsilon_{ij} \]
      for \( \beta_2 = \beta_1 \omega \)
      i. Larger model includes interaction between preparation and slope, or, equivalently, different slopes for each preparation
      ii. Failure of smaller model implies that potency is not constant across dose levels.
      iii. One can test this.
   f. Typically assume \( \epsilon_{ij} \) (ie., errors on log scale) are i.i.d. normal
      i. Normality not so important for reasonably large data set
   g. Apply techniques for the pooled–variance–estimate case to bound \( \omega = \beta_2 / \beta_1 \).
   h. Technique is called parallel line bioassay.
   i. Choice of metameter affects interpretation of study.
   j. Choice of transformation to response is less important, since model correctness implies equality on response scale of new preparation with standard preparation at potency times new dose, for all doses.

E. Quantal Assay
1. Similar to direct assay
   a. Two preparations to be compared
   b. Except that response is proportion responding, rather than a quantitative response
2. Aim remains
   a. to find dose yielding a fixed response level
      i. Common symbol: ED (effective dose) or LD (lethal dose) with subscript giving percent \( 100\pi \) of individuals responding
         \[ \log(\pi) - \log(1 - \pi) = \beta_0 + \beta_1 \xi \iff \xi = [(\log(\pi) - \log(1 - \pi)) - \beta_0] / \beta_1. \]
         i. estimated by \( \hat{\xi} = [(\log(\pi) - \log(1 - \pi)) - \hat{\beta}_0] / \hat{\beta}_1. \)
      ii. For ex., \( \log(.75) - \log(.25) = 1.0986 \).
   b. to find multiplier to make response of one treatment equivalent to another
      i. Binary responses (usually)
      ii. Logistic or probit link, or some other dose metameter
      iii. Text suggests adjustment of variance in response to change in fitted value via variance–stabilizing rather than standard re-weighting for generalized linear model.
         i. I can’t figure out why.
   iv. Then as before, \( Y_{ij} \sim \text{Bin}(\pi_{ij}, n_{ij}) \)
      \[ \pi_{ij} = \beta_0 + \beta_1 x_{ij} + \beta_2 i \] for \( i = 0, 1 \)
      i. Generally \( x_{ij} \) represents log dose
      ii. As before, potency is given by \( \beta_2 / \beta_1 \)
      iii. CI given by Fiellier’s method
         ▶ Known variance case