Note: The problems are taken from the Exercises in Casella and Berger (2002) unless otherwise noted. For each problem, please explain your reasoning clearly. It is not acceptable to only provide your final result.

Homework 1 (Due Wed, Sept 18):
6.9, 6.10, 6.11, 6.12, 6.15, 6.20, 6.23, 6.30, 6.31(b,c)

The question 6.31(b)(ii) should be corrected as follows. Suppose \( X^1, \ldots, X^N \) are \( N \) Monte Carlo samples, where each \( X^j \) consists of an iid sample of size \( n \) from \( N(\mu, \sigma^2) \). Let \( M^j \) be the sample median and \( \bar{X}^j \) the sample mean, both computed from \( X^j \). Let \( \bar{M} \) be the average of \( (M^1, \ldots, M^N) \), and \( \bar{\bar{X}} \) be the average of \( (\bar{X}^1, \ldots, \bar{X}^N) \). A naive estimator of the variance of the sample median is then

\[
v_1 = \frac{1}{N-1} \sum_{j=1}^{N} (M^j - \bar{M})^2.
\]

The swindle estimator of the variance of the sample median is

\[
v_2 = \frac{\sigma^2}{n} + \frac{1}{N-1} \sum_{j=1}^{N} \{M^j - \bar{X}^j - (\bar{M} - \bar{\bar{X}})\}^2.
\]

Show that the variance of \( v_1 \) is approximately \( 2[\text{var}(M)]^2/(N-1) \), and the variance of \( v_2 \) is approximately \( 2[\text{var}(M - \bar{X})]^2/(N-1) \).

Additional exercise (optional): Read the paper, “Models as Approximations, Part I: A Conspiracy of Random Regressors and Model Deviations Against Classical Inference in Regression” by Buja et al., currently available at https://www.imstat.org/journals-and-publications/statistical-science/statistical-science-future-papers/. Think about three questions you may have about the paper.