

RUTGERS UNIVERSITY  
DEPARTMENT OF STATISTICS AND BIostatISTICS

501 Hill Center, Busch Campus  
110 Frelinghuysen Road  
Piscataway NJ 08854-8019

[www.stat.rutgers.edu](http://www.stat.rutgers.edu)

**Speaker:** Kevin Ross  
Stanford University  
[kjross@stanford.edu](mailto:kjross@stanford.edu)

**Title:** Optimal Stopping and Free Boundary Characterizations for some Brownian Control Problems

**Date:** Wednesday, October 24, 2007

**Time/Place:** 3:20 PM, in 552 Hill Center, Busch Campus

**Abstract**

We study a singular stochastic control problem with state constraints in two-dimensions. We show that the value function is continuously differentiable and its directional derivatives are the value functions of certain optimal stopping problems. Guided by the optimal stopping problem we then introduce the associated no-action region and the free boundary and show that, under appropriate conditions, an optimally controlled process is a Brownian motion in the no-action region with reflection at the free boundary. This proves a conjecture of Martins, Shreve and Soner (*SIAM J. Control Optim.* **34** (1996), no. 6, 2133--2171) on the form of an optimal control for this class of singular control problems. An important issue in our analysis is that the running cost is Lipschitz but not continuously differentiable. This lack of smoothness is one of the key obstacles in establishing regularity of the free boundary. We show that the free boundary is Lipschitz and if the Lipschitz constant is sufficiently small, a certain oblique derivative problem on the no-action region admits a unique viscosity solution. This uniqueness result is key in characterizing an optimally controlled process as a reflected diffusion in the no-action region. (Joint work with Amarjit Budhiraja.)

**Refreshments:** 2:50 PM in 502 Hill Center