RUTGERS UNIVERSITY
DEPARTMENT OF STATISTICS AND BIOSTATISTICS
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Seminar

Speaker: Larry Shepp
Statistics Department, Rutgers University

Title: Open Problems of Current Interest to Me

Date: Wednesday, April 2, 2008

Time: 3:20 PM

Place: 552 Hill Center

Abstract

I will talk about 4 problems, time permitting, the first statistical, the other 3 are probabilistic and tied through characteristic functions.

I. There are two new devices which hold promise to alleviate the tedium of managing a diabetic - see http://www.youtube.com/watch?v=BDATgiMwRNA for a quick understanding of the tedium. The first is the insulin pump; the second is the continuous glucose sensor. The problem is to merge them into a closed loop system. Many open problems remain.

A characteristic function (chf) of a probability dist is
$\phi_X(z) = Ee^{iXz} = \int e^{ixz} F(dx)$,
where $F(a) = F_X(a) = P(X \leq a)$ is the cdf of a r.v. X. If $X \sim -X$, X is called symmetric and ϕ is real valued (and conversely).

II. Find the most general stationary Gaussian process, $Y(t), t \in \mathcal{R}$, such that the vector process $(Y(t), Y'(t), \ldots, Y^{(n)}(t))$ is Markov. Joint with Lazy Brown et al.

III. e^{-z2} is a chf but just barely so. Marcinkiewicz's theorem says that $e^{P(z)}$ is not a chf if P is a polynomial of degree > 2. A new question, due to Aiyou Chen, asks whether $f(z)e^{-z4}$ is a chf for any analytic chf f.

IV. Let $X_n(t) = \sum_{k=0}^{n} \xi_j t^j$ be a random polynomial with iid symmetric coefficients, ξ_j. How many real zeros, N_n, does it have?

Kac-Rice formula:
$EN_n = \frac{4}{\pi 2} \int_0 1 \frac{dt}{t} \int_0^\infty \frac{du}{u} \int_0^\infty dv [2 \prod_{j=0}^{n} \phi(ut^j) - \prod_{j=0}^{n} \phi(ut^j(1+\frac{j}{v})) - \prod_{j=0}^{n} \phi(ut^j(1-\frac{j}{v}))]$

If ξ is in the domain of attraction of the symmetric stable law with chf e^{-z^α}, then the mean number, EN_n, of real zeros is $c(\alpha) \log n$ as $n \to \infty$, where $c(\alpha)$ is explicitly given and decreases from 1 to $\frac{2}{\pi}$ as α increases from 0 to 2. This suggests that as the tails of ξ get larger and larger the number of real zeros increases, but a recent theorem of Dimitrii Zaparozhets says that there are really long tailed ξ with a bounded number of real zeros. How to see the mean number of real zeros turn around for ever larger tailed Polya type distributions from the Kac-Rice formula?