Impact of measurement error on container inspection policies at port-of-entry

Yada Zhu · Mingyu Li · Christina M. Young ·

Minge Xie · Elsayed A. Elsayed

Abstract Containers and cargos arriving at port-of-entry are inspected using sensors and devices to detect drugs, weapons, nuclear materials and other illegal items. Measurement errors associated with the inspection process may result in higher percentage of misclassification of containers. In this paper, we propose and formulate three inspection policies for containers at port-of-entry assuming the presence of sensor measurement errors. The optimization of the policies is carried out and the performance of each in terms of misclassification probabilities is compared. In each of the policies, the optimum settings are determined by minimizing the probability of false rejection while limiting the probability of false acceptance at a very low tolerance level. The results show that the policy of repeat inspections improves the performance in terms of correct container classification.

Keywords POE · Container inspection · Measurement error · Sensor threshold · Re-inspection band · Container misclassification

1 Introduction

Global commerce is totally dependent on the movement of shipping containers, which carry about 95 percent of the world's international cargo in terms of value. Containers carry a wide range of materials, food, equipment and other types of products and commodities. They may also transport drugs, arms, chemical, nuclear and biological materials, and operatives for illegal activities, yet fewer than two percent of them are subject to in-depth inspection (Dahlman, *et al*. 2005).

Disruption of the maritime shipping due to risk associated with its contents would have a profound effect on the world economy. Moreover, the increasing dependence of companies on just-in-time deliveries of raw material and components, and the global distribution of its products and extensive supply chain networks between suppliers and companies magnifies the impact of the security issues of maritime shipping. This has prompted the investigation and implementation of different procedures for ensuring container securities. These range from assessing container risk (security) beginning from the shipping origin to inspection at destination. For example, the National Defense University Center for Technology and National Security Policy recognizes that the risk associated from seaborne containers bound for the United States begins at the point of origin, which should also be the point of inspection. Beginning U.S. control over cargo at the foreign point of origin would create a "virtual border," a multi-layered defense addressing container security from the initial loading of the container to its movement through the entire

This research is conduction with partial support from ONR grant numbers N00014-05-1-0237, N00014-07-1-029, NSF grant number NSFSES 05-18543 and NSA grant number H98230-08-1-0104 to Rutgers University

Y. Zhu · C. M. Young · E. A. Elsayed (✉)
Department of Industrial and Systems Engineering. Rutgers University, Piscataway, NJ, 08854, USA
e-mail: elsayed@rci.rutgers.edu

Y. Zhu
email: yadazhu@eden.rutgers.edu

C.M. Young
email: cms168@rci.rutgers.edu

M. Li · M. Xie (✉)
Department of Statistics, Rutgers University, Piscataway, NJ, 08854, USA
email: mxie@stat.rutgers.edu

M. Li
email: limingyu@eden.rutgers.edu

international transportation network. The concept of a virtual border formed the foundation for the Container Security Initiative adopted by the United States bilaterally with a number of other countries (Dahlman, *et al*. 2005).

In addition to assessing the container risk at its origin port, the inspection of containers at ports-of-entry (POE) is critical for detecting and preventing illegal cargo from entering the United States. The inspection process can be generalized as the collection and analysis of information attained from the application of different types of detectors such as radiation, biological and chemical detectors, sensor instruments or other types of inspection such as manual inspection to decide whether to allow a container to pass through the port. Clearly, the accuracy of inspection in terms of passing (or accepting) those containers which indeed do not have illegal cargo and rejecting those which indeed contain such cargo with minimum delays depends on many factors such as the acceptance threshold levels of the instrumentations (sensors), accuracy and precision of the instruments and others. Therefore, formulating a mathematical model of the container inspection allows for evaluation and improvement of the process. The POE inspection process has been investigated by Elsayed *et al.* (2008), Boros *et al*. (2008), Ramirez-Marquez (2008) and Wein *et al*. (2006). These papers consider the inspection process as a sequence of sensors (instruments or equipment), each is dedicated to the detection of a specific characteristic of the undesirable material in the container. For instance, gamma-ray is used for the detection of radioactive material while x-ray imaging is used to for the analysis of images of unusual (unexpected) contents of the container and the biological instruments are used to detect biological agents. The models investigated focus on the optimization of the inspection process by determining the threshold levels and sequence of inspection stations. The terms inspection station, sensor, and device are used interchangeably in this paper. A collection of stations forms an inspection system.

Brandenstein (2007) provides the highlights of the role risk assessment played in the United States technology program for nonintrusive inspection of cargo containers for illicit drugs. Koch (2007) develops a port simulation model to investigate the effect of the introduction of new inspection technologies on the overall port operations.

In modeling the POE inspection systems, the investigators seek the optimum threshold levels of the specific container characteristics (such as acceptable radiation level) and the optimum sequence of inspection that minimize the total cost or inspection time in the system. Dye (2003) summarizes some basic requirements of inspection systems: 1. Sensor systems must be operationally practical and must provide information that can enable effective, preemptive actions to be taken, 2. Sensors systems must be highly sensitive, providing a low probability of missed detections (false negatives) and 3. Sensor systems must give a low probability of false alarms (false positives).

No sensor system can provide a “perfect” solution to these competing requirements; the best that can be done is a compromise that strikes a balance among all three. Such a compromise may entail a layered defense, exploiting a combination of complementary sensor types. By sensing different characteristics at successive layers, the system-wide count of false negatives is greatly reduced, as one sensor’s strengths can be used to offset another’s weaknesses. Likewise, successive layers help reduce false positives. Another approach is to minimize the impact of the sensor errors (measurement errors) linked with appropriate characteristic threshold levels. Large measurement errors may result in significant misclassifications of the containers (false positive and false negative). Therefore, one needs to minimize the impact of such errors (Mader *et al.* 1999) on the accuracy of container classifications. This can be achieved by considering the measurement error in the inspection model and/or the development of effective inspection policies.

There are many sources that contribute to measurement errors which include human errors, gauge errors and environments. It is important that the sources are identified and their contributions are reduced. In many cases, it is difficult, if not impossible to do so. However, researchers have utilized two approaches in order to minimize the effect of measurement errors (Kim *et al*. 2007): the first approach deals with the reduction of variability of the measurements through the use of more precise measurement devices and/or better-trained operators (Chandra and Schall 1988, Chen and Chung 1996, Tang 1988). The second approach is based on the use of guard bands to identify “good” and “bad” items. The economic impact of guard bands is investigated by Eagle (1954), Grubbs and Coon (1954) and Hutchinson (1991). Deaver (1995) provides a comparative study of several strategies for the use of guard bands. McCarville and Montgomery (1996) develop an experimental design approach for finding the optimal guard bands for serial gauges. Recently, Kim *et al.* (2007) integrate these two approaches and develop an optimization scheme for the design of the most economical measurement procedures that simultaneously determine both the optimum precision level and guard band in order to reduce the impact of measurement errors.

This paper considers approaches to reduce the impact of measurement errors in the POE inspection policies. More specifically, Elsayed *et al.* (2008) develop a model of the inspection system and illustrate how an inspection policy can be optimized. However, the model uses a single random variable to represent the sensor reading and does not consider sensor measurement error independently from the natural variation in the container attribute values. The measurement error associated with inspection devices has a significant effect on the inspection decisions, and taking this into account would improve the model's accuracy. We consider realistic situations where measurement errors exist (and embedded) in the readings obtained by the inspection devices. When a simple accept/reject threshold is used, containers with readings close to the threshold value are at risk for misclassification. Therefore, we investigate and optimize container inspection policies under different inspection strategies involving repeat inspections.

The paper is organized as follows. Sect. 2 describes the port-of-entry inspection problem and an associated measurement error model. This section also introduces three inspection policies and corresponding optimization problem. Sect. 3 presents the mathematical formulation of the inspection policies proposed in this paper. Sect. 4 compares the performance of each policy by numerical examples. Sect. 5 extends each policy from single inspection station to a system and finally the last section offers conclusion and discussion of the present work.

2 Problem descriptions

2.1 Container inspection

Containers arriving to a port-of-entry are inspected to prevent entry of undesired cargo such as illegal weapons, drugs, and dangerous material. Each container has several attributes and the presence of one or more of the attributes may lead to additional inspection that may require examining the contents of the container manually. The attributes may include radioactive material, biological and chemical agents, drugs and illegal weapons. A typical inspection system may consist of several stations; each inspects one particular attribute of the container. Elsayed *et al.* (2008) consider an inspection system as a collection of *n* stations where the inspection for the presence of the attributes in a given container is performed sequentially. At each station the decision to accept or reject a container is dependent on preset threshold levels corresponding to these attributes. The overall container classification of "good" or "bad" is based on a system decision function *F* that assigns to each binary string of decision $\left(d_1, d_2, \ldots, d_n\right)$ a category (0 or 1). Here, *F* is a Boolean function which is constructed based on the inspection sequence of the container and the decision function of the system. Elsayed *et al.* (2008) simultaneously determine the optimum sequence of inspection or the structure of the inspection decision trees, and the optimum thresholds of the inspection stations that minimize the total inspection cost. In this paper, we follow the same inspection process and consider a more realistic situation where the sensor or station readings include measurement errors. We begin with a simple model and consider one station that inspects only one attribute. A preset threshold level *T* is used: if a reading '*y*' is greater than the threshold level the attribute fails inspection, and if $y \leq T$ then it passes. It is clear that the station decision is dependent on the preset *T* and the measurement error. We then extend the work to inspection systems with multiple stations.

2.2 Sensor modeling with measurement error

Let *x* represent the true status of a particular container attribute, such that a container with $x = 0$ should be accepted and one with $x = 1$ should be rejected. Over the entire container population the probability of $x = 1$ is $P(x = 1) = \pi$ and the probability of $x = 0$ is $P(x = 0) = 1 - \pi$. Since the true attribute reading *y* is dependent on *x*, following Stroud and Saeger (2003) and Elsayed *et al.* (2008), we assume two different distributions: $\left(y \mid x = 0\right) \sim N\left(\mu_0, {\sigma_0}^2\right)$ and $\left(y \mid x = 1\right) \sim N\left(\mu_1, {\sigma_1}^2\right)$. We choose to use the normal distribution because normally distributed data are the most commonly seen in practice and this assumption has been used in port-of-entry inspection applications, such as Stroud and Saeger (2003), Elsayed *et al*. (2008) and others. In this paper, we further assume that the measurement taken by the sensor, *r*, is affected by both the true attribute reading y and some small random measurement error •, $r = y + \varepsilon$. Here y and • are independent. So if we assume the measurement errors are distributed normally, $\varepsilon \sim N\left(0, {\sigma_\varepsilon}^2\right)$, then the distributions of the observed readings from the sensor can be written as $\left(r \mid x = 0\right) \sim N\left(\mu_0, {\sigma_0}^2 + {\sigma_\varepsilon}^2\right)$ and $\left(r \mid x = 1\right) \sim N\left(\mu_1, {\sigma_1}^2 + {\sigma_\varepsilon}^2\right)$.

When a simple threshold is applied to make a decision, there is a chance to accept an unacceptable container or reject an acceptable one, called misclassification error. The presence of measurement error contributes to this misclassification error. One way to decrease the impact of the sensor measurement error on misclassification is to repeat measurements with the same sensor. Multiple measurements (*r* values) taken by a given sensor have the same *y* value but different • values and since *y* and • are independent, averaging these measurements provides a more accurate estimate compared to a single measurement. In light of this, inspection policies can be developed that apply repeated measurements (referred to as repeat inspections) for selected containers. The method for selecting containers for repeat inspection should identify ambiguous containers at higher risk for misclassification. This can be achieved by applying a "re-inspection band" *b* around a T so that containers with $T - b/2 \le r \le T + b/2$ are subject to additional inspections. The concept for using a re-inspection band in the selection of containers for repeat inspection is related to the use of guard bands. However, different from the use of guard bands in quality control literature, we do not accept containers within the band limits in the POE inspection practice but they are rather selected for further inspection.

2.3 Inspection policies

Consider the simple inspection process mentioned in Sect. 2.1, in which a sensor reading *r* is compared to a specified threshold value *T*, returning an "accept" decision $d = 0$ if *r* • *T* and a "fail" decision $d = 1$ if *r* > *T*. In this decision making process, there is a chance to reject an acceptable container (true status $x = 0$). This misclassification error is the probability of false rejection (*PFR*). There is also a chance to accept an unacceptable container (true status $x = 1$). This error is the probability of false acceptance (*PFA*). The presence of the random measurement error term increases both the variability of the measurements and misclassification of containers in the inspection process. Fig. 1 shows the effect of measurement error on the *PFR* and *PFA* values of an inspection system. The probability of false rejection is represented by the area to the right of *T* under the solid line $x = 0$, which is less than the analogous area under the dotted line $r \mid x = 0$ (highlighted with horizontal stripes). This difference corresponds to the increase in *PFR* when measurement error is included in the sensor reading *r*. Similarly, the area to the left of *T* under the solid line $x = 1$ represents the probability of false acceptance, which expands to the area under the dotted line $r \mid x = 1$ (highlighted with vertical stripes) when there is measurement error associated with the sensor reading *r*.

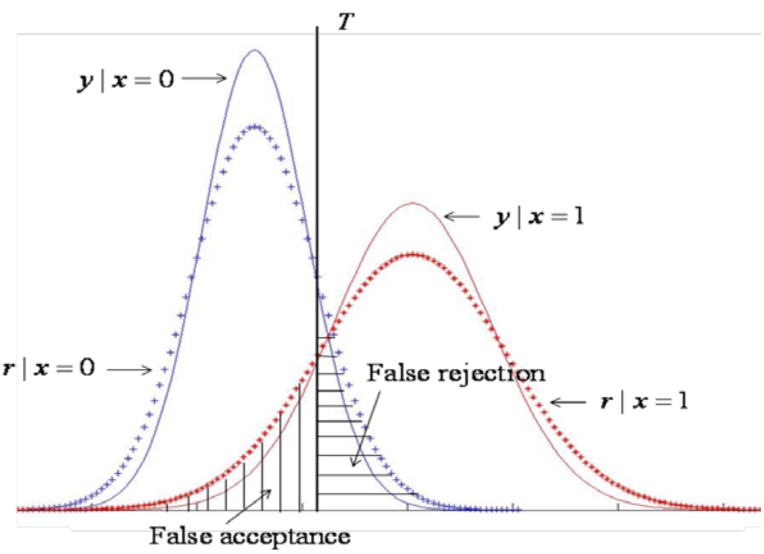

Fig. 1 Probability of false acceptance and false rejection

Policy I is a simple inspection process of comparing sensor readings to a preset threshold level. Measurement values are obtained from a sensor and containers with r • T are accepted, while those with $r > T$ are rejected. The value of T which is given as part of the policy has a significant effect on the performance of the inspection station and therefore is used as a decision variable in the optimization of Policy I. This policy is simple and inexpensive to apply, however it is not adjusted for measurement error inherent in the measurements.

The presence of measurement error can have a significant effect on the performance of the inspection process, as illustrated in Fig. 1. Although unavoidable in many cases, random measurement error is often ignored in theory and practice due to a variety of reasons. As mentioned in Sect. 2.2, repeat inspections are one way to improve performance and reduce misclassification by effectively decreasing the error through the collection of independent measurements. Two inspection policies that use repeat inspections to address the problem of measurement errors are proposed and their performances are compared with Policy I.

In manufacturing and production environment, to satisfy certain quality control criterion and also to avoid the high penalty cost of mistakenly accepting non-conforming units, the units are often subjected to 100 percent repeat inspection. This can reduce the effects of measurement errors in terms of *PFA* and *PFR*. However, such high percentage of repeat inspection in the port-of-entry problem is likely to increase waiting time and delay the delivery of containers. From Fig. 1, containers with readings close to the threshold T are at high risk for misclassification and applying repeat inspection to those containers will reduce the probability of misclassification. Therefore, repeat inspection is conducting in the following policies using a re-inspection band of width b around the threshold T, which can both be adjusted for optimal performance. We integrate the re-inspection band and repeat inspection into two inspection policies to reduce the impact of measurement errors on the false classification rate.

Policy II is an example of using repeat inspection to decrease the measurement error when a more precise inspection sensor is unavailable. The first step of this two-step process is to apply a re-inspection band b to create a range around T and assess measurement values from an initial inspection so that containers with readings falling above or below these limits are rejected or accepted, respectively. Fig 2 illustrates the effect of the re-inspection band on *PFR* and *PFA* in decisions made in the first step of Policy II. Comparing Fig. 1 and 2, it is obvious that the *PFA* and *PFR* of these decisions falling outside the re-inspection band applied around T are less than the *PFA* and *PFR* of Policy I.

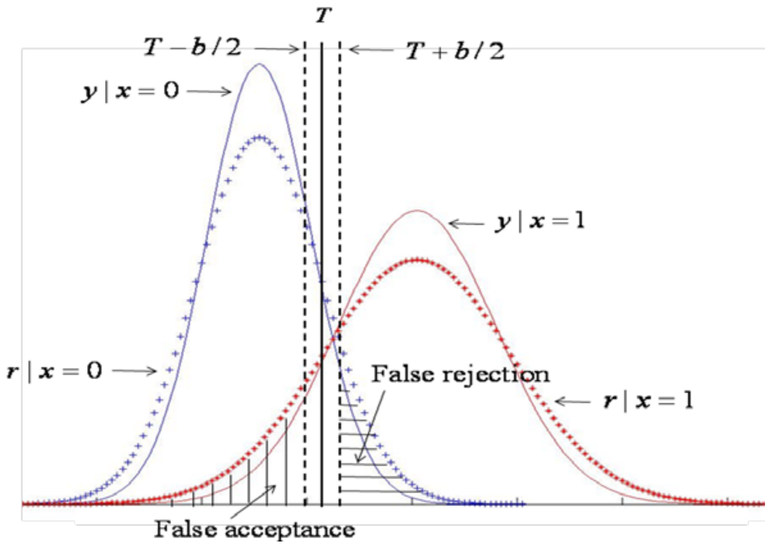

Fig. 2 Re-inspection band and misclassification error

Containers with initial readings falling in $T - b/2 \leq r \leq T + b/2$ are selected for the second step of Policy II, in which the inspection is repeated n times with the same sensor and the average reading is used to provide a more accurate measurement. This average is then compared to T for a final accept/reject decision. The decision variables in this policy are b and T.

Taking the average of n repeat measurements reduces the variability of the measurement readings. An alternative approach to control the measurement error is to select sensors with greater precision as described in Policy III below. It should be noted that an inspection process with greater precision usually incurs higher inspection cost due to the requirement of advanced equipment, better trained operators and well controlled environment, etc.

The third policy is capable of improving performance over Policy II. Policy III uses more precise sensors in repeat measurements, which reduce the impact of measurement errors. We assume that a series of measurements can be obtained, each with improved accuracy. This could be achieved either by adjusting the operational settings of one sensor, using different sensors, or some combination of the two. Policy III has an identical first step to Policy II, where initial readings are compared with the re-inspection band to make decisions to accept, re-inspect when the readings are within the band, or reject containers. The repeat inspection is done with sensor with improved precision, ${\sigma_{R\varepsilon}}^2 < {\sigma_\varepsilon}^2$. This process of inspecting with an improved sensor and applying a re-inspection band can be repeated *n* times, at which time a final measurement is taken and compared directly with *T* for an accept/reject decision, as in Policy I.

To find the optimal settings for these policies, we must first define an objective function. *PFA* and *PFR* are useful measures of inspection performance. Since a minimum requirement is more often specified for *PFA*, we define the objective function for all policies as $\min PFR$ subject to $PFA \le FA^*$, where FA^* is a specified upper limit. Objective functions for each policy are formulated in the following section.

3 Formulation of inspection policies

3.1 Policy I

Policy I is a simple inspection process of comparing each sensor reading *r* against a preset decision threshold *T* to make a pass or fail decision. If *r* > *T*, a reject decision *d* = 1 is made, otherwise, an accept decision *d* = 0 is made. According to the additive error model, $r = y + \varepsilon$, and assumptions of the distribution of measurement readings, $(r \mid x = 0) \sim N\left(\mu_0, {\sigma_0}^2 + {\sigma_\varepsilon}^2\right)$ and $(r \mid x = 1) \sim N\left(\mu_1, {\sigma_1}^2 + {\sigma_\varepsilon}^2\right)$ for acceptable and unacceptable containers respectively, the *PFR* and *PFA* for Policy I are given by:

$$PFR = P(d = 1 \mid x = 0) = 1 - \Phi\left(\frac{T - \mu_0}{\sqrt{\sigma_0^2 + \sigma_\varepsilon^2}}\right)$$

$$PFA = P(d = 0 \mid x = 1) = \Phi\left(\frac{T - \mu_1}{\sqrt{\sigma_1^2 + \sigma_\varepsilon^2}}\right)$$

The optimization of Policy I is determined by minimizing the probability of false rejection while limiting the probability of false acceptance at some low level, which is often defined by a requirement. This optimization is formulated as $\min PFR$ subject to $PFA \le FA^*$. The decision variable for this policy is *T*.

3.2 Policy II

Policy II is a two step process which employs a re-inspection band and repeat inspections in an attempt to reduce the impact of measurement error. In the first step, the inspection reading r_1 of an attribute is obtained and compared against the limits of the re-inspection band *b* placed around a given *T*. Containers with $r_1 < T - \frac{b}{2}$ are accepted and containers with $r_1 > T + \frac{b}{2}$ are rejected. The remaining containers with r_1 values that fall within the re-inspection band $\left(T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\right)$ proceed to the next step, in which they are subject to $l-1$ repeat inspections with the same sensor (*l* is given). Let $r_2, ..., r_l$ be the observed values from the $l-1$ repeat inspections, and let $\overline{r}_{2-l}$ be their average value. If $\overline{r}_{2-l} \le T$, the container is accepted; if $\overline{r}_{2-l} > T$, the container is rejected. The distribution of the average reading $\overline{r}_{2-l}$ is given by

$\overline{r}_{2-l} \mid x = i \sim N\left(u_i, \sigma_i^2 + \sigma_\varepsilon^2 / (l-1)\right)$ for $i = 0,1$.

The total *PFR* of Policy II is

$$P\{D=1 \mid x=0\} = P\{d_1 = 1 \mid x=0\} + P\left\{d_{2-l} = 1 \mid x=0, T-\frac{b}{2} \le r_1 \le T+\frac{b}{2}\right\} P\left\{T-\frac{b}{2} \le r_1 \le T+\frac{b}{2} \mid x=0\right\}$$

$$= 1 - \Phi\left(\frac{T+\frac{b}{2}-\mu_0}{\sqrt{\sigma_0^2+\sigma_\varepsilon^2}}\right) + \int_{-\infty}^{\infty}\left\{1-\Phi\left(\frac{T-y}{\sigma_\varepsilon/\sqrt{l-1}}\right)\right\}\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_\varepsilon}\right) - \Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_\varepsilon}\right)\right\}\frac{\phi\left(\frac{y-\mu_0}{\sigma_0}\right)}{\sigma_0}dy$$

and the total *PFA* of Policy II is

$$P\{D=0 \mid x=1\} = P\{d_1 = 0 \mid x=1\} + P\left\{d_{2-l} = 0 \mid x=1, T-\frac{b}{2} \le r_1 \le T+\frac{b}{2}\right\} P\left\{T-\frac{b}{2} \le r_1 \le T+\frac{b}{2} \mid x=1\right\}$$

$$= \Phi\left(\frac{T-\frac{b}{2}-\mu_1}{\sqrt{\sigma_1^2+\sigma_\varepsilon^2}}\right) + \int_{-\infty}^{\infty}\Phi\left(\frac{T-y}{\sigma_\varepsilon/\sqrt{l-1}}\right)\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_\varepsilon}\right) - \Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_\varepsilon}\right)\right\}\frac{\phi\left(\frac{y-\mu_1}{\sigma_1}\right)}{\sigma_1}dy,$$

where D denotes the final decision, d_1 denotes the decision in the first inspection and d_{2-l} denotes the decision after re-inspection in the second step. Similar to the Policy I, the optimization problem of Policy II can be formulated as $\min PFR$ subject to $PFA \le FA^*$. The decision variables are *T* and *b*.

3.3 Policy III

Policy III assumes that the measurement errors can be controlled and decreased in subsequent inspections. In the first step, the inspection reading r_1 of an attribute is obtained and compared against the limits of the re-inspection band *b* placed around a given *T*. Let the variance of measurement error associated with the first inspection be designated $\sigma_{\varepsilon 1}{}^2$. Containers with $r_1 < T - \frac{b}{2}$ are accepted and containers with $r_1 > T + \frac{b}{2}$ are rejected. The remaining containers with r_1 values that fall within the re-inspection band $\left(T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\right)$ proceed to the next step where they are inspected and return an observed value r_2. This second inspection is subjected to a smaller variance in measurement error, $\sigma_{\varepsilon 2}{}^2 < \sigma_{\varepsilon 1}{}^2$ corresponding to sensors with higher level of precision. If $r_2 < T - \frac{b}{2}$, the container is accepted; if $r_2 > T + \frac{b}{2}$, the container is rejected. If $T - \frac{b}{2} \le r_2 \le T + \frac{b}{2}$, the container is then inspected by yet a more precise sensor with smaller variance in measurement error, $\sigma_{\varepsilon 3}{}^2 < \sigma_{\varepsilon 2}{}^2$ and the observed value from this third inspection is designated r_3. This process of taking readings and filtering three ways (accept, reject, re-inspect) is repeated $l-1$ times (*l* is given) to get $r_1, ..., r_{l-1}$. In the last step, the l^{th} inspection is performed and the reading is compared against *T*; if $r_l \le T$ the container is accepted and if $r_l > T$ the container is rejected.

For Policy III, the total *PFR* over *l* inspections is

$$P\{D=1 \mid x=0\} = P\{d_1 = 1 \mid x=0\}$$

$$+\sum_{k=2}^{l} P\left\{d_k = 1 \mid x=0, T-\frac{b}{2} \le r_j \le T+\frac{b}{2}, 1 \le j < k\right\} P\left\{T-\frac{b}{2} \le r_j \le T+\frac{b}{2}, 1 \le j < k \mid x=0\right\}$$

$$= 1-\Phi\left(\frac{T+\frac{b}{2}-\mu_0}{\sqrt{\sigma_0^2+\sigma_{\varepsilon 1}^2}}\right) + \sum_{k=2}^{l-1}\int_{-\infty}^{\infty}\left\{1-\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_{\varepsilon k}}\right)\right\}\prod_{j=1}^{k-1}\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)-\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)\right\}\frac{\phi\left(\frac{y-\mu_0}{\sigma_0}\right)}{\sigma_0}dy$$

$$+\int_{-\infty}^{\infty}\left\{1-\Phi\left(\frac{T-y}{\sigma_{\varepsilon l}}\right)\right\}\prod_{j=1}^{l-1}\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)-\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)\right\}\frac{\phi\left(\frac{y-\mu_0}{\sigma_0}\right)}{\sigma_0}dy$$

and the total *PFA* over *l* inspections is

$$P\{D=0 \mid x=1\} = P\{d_1 = 0 \mid x=1\}$$

$$+\sum_{k=2}^{l} P\left\{d_k = 0 \mid x=1, T-\frac{b}{2} \le r_j \le T+\frac{b}{2}, 1 \le j < k\right\} P\left\{T-\frac{b}{2} \le r_j \le T+\frac{b}{2}, 1 \le j < k \mid x=1\right\}$$

$$= \Phi\left(\frac{T-\frac{b}{2}-\mu_1}{\sqrt{\sigma_1^2+\sigma_{\varepsilon 1}^2}}\right) + \sum_{k=2}^{l-1}\int_{-\infty}^{\infty}\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_{\varepsilon k}}\right)\prod_{j=1}^{k-1}\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)-\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)\right\}\frac{\phi\left(\frac{y-\mu_1}{\sigma_1}\right)}{\sigma_1}dy$$

$$+\int_{-\infty}^{\infty}\Phi\left(\frac{T-y}{\sigma_{\varepsilon l}}\right)\prod_{j=1}^{l-1}\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)-\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_{\varepsilon j}}\right)\right\}\frac{\phi\left(\frac{y-\mu_1}{\sigma_1}\right)}{\sigma_1}dy$$

where D denotes the final decision and d_k denotes the decision after the k^{th} inspection. Similar to Policy I and II, the optimization of Policy III is formulated as $\min PFR$ subject to $PFA \le FA^*$. Assuming the sequence of decreasing $\sigma_\varepsilon{}^2$ values is given, the decision variables in this policy are *T* and *b*.

4 Numerical examples

Numerical examples are presented in this section to demonstrate the effectiveness of the proposed inspection policies in a single station. Note that in the port-of-entry inspection problem, the goal is to separate two groups of containers and the problem is location and scale invariant (Elsayed *et al*. 2008). Without loss of generality, we choose $\mu_0 = 0$, $\mu_1 = 1$ and some reasonable values for the associated standard errors. In the numerical example, we use $\sigma_0 = 0.35$, $\sigma_1 = 0.1$. Since in practice inspections can only be repeated for a limited number of times, the total number of inspections for Policy II and III is limited to three. If a container is suspicious after inspections (its status is not defined), the container is subjected to manual inspection as it actually occurs in the port. The magnitude of measurement error in the initial inspection is assumed to be $\sigma_\varepsilon = 0.06$ in the numerical example. The magnitude of measurement error for the second and third inspections of Policy III are $\sigma_{\varepsilon 2} = 0.03$ and $\sigma_{\varepsilon 3} = 0.015$, respectively. Matlab is used to determine the optimal solution by implementing a constrained nonlinear multivariable algorithm, *fmincon*. In the optimization, the upper and lower limits of the threshold value and re-inspection band width are limited within 0 and 1. The objective function is to minimize *PFR,* subject to a *PFA* constraint of 10^{-3}, 10^{-4}, and 10^{-5}, respectively for the three policies. The minimum *PFR* values with respect to different *PFA* constraints for each policy are presented in Fig. 3. The associated optimal threshold and the band width are shown in Table 1.

It is evident from Fig. 3 that Policy II consistently returns smaller *PFR* than Policy I, and Policy III consistently returns smaller *PFR* than Policy II under the same constraints. This indicates that the inspection process can be improved by application of re-inspection band and repeat inspections, and further improved if the repeat inspections are conducted by sensors with higher precision levels.

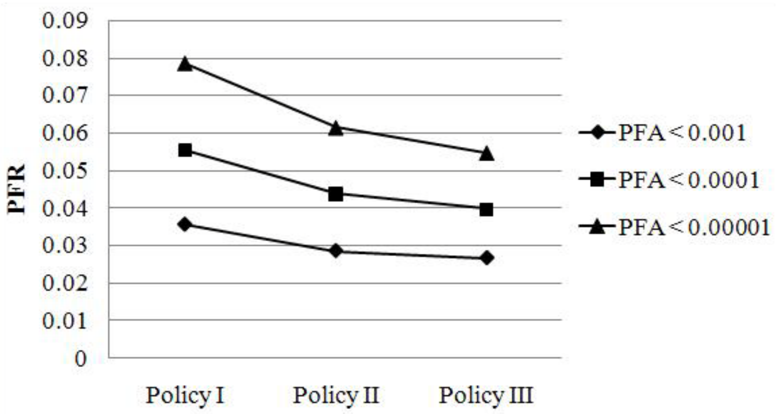

Fig. 3 Minimum *PFR* obtained from each policy subject to different constraints

Table 1 Optimal threshold value and band width associated with minimum *PFR*

PFR / *PFA* <	Policy I	Policy II		Policy III	
	Threshold	Band Width	Threshold	Band Width	Threshold
0.001	0.6396	0.1539	0.6689	0.2379	0.687
0.0001	0.5663	0.1594	0.6015	0.2901	0.6238
0.00001	0.5026	0.1633	0.5429	0.3058	0.5686

As Fig. 3 indicates, both Policy II and III can improve the inspection process in terms of *PFR* under each constraint. The percentages of improvement are investigated and depicted in Table 2. The results listed in column 1, 2, and 3 of Table 2 are the percentage of reduction of *PFR* values defined by $\frac{(\text{Policy I-Policy II})}{\text{Policy I}} \cdot 100\%$, $\frac{(\text{Policy I-Policy III})}{\text{Policy I}} \cdot 100\%$, and $\frac{(\text{Policy II-Policy III})}{\text{Policy II}} \cdot 100\%$, respectively. Comparing the values in column 2 and 3 row by row, it is noticeable that the improvement of Policy III over Policy I is always larger than that of Policy II over Policy I. Mean while, the last column of Table 2 shows the improvement of Policy III over Policy II, all of which are positive values. These results coincide with our intuition that Policy III, which uses sensors with higher precision level on each conditional re-inspection, should perform better than the other policies.

Table 2 Percentage of improvement

PFR / *PFA* <	Policy I vs. Policy II	Policy I vs. Policy III	Policy II vs. Policy III
0.001	19.83%	25.14%	6.62%
0.0001	20.94%	28.16%	9.13%
0.00001	21.66%	30.19%	10.89%

The discussion above confirms that repeat inspection can reduce the negative effects of measurement errors and repeat inspection by sensors with increased precision level can further reduce the effects of measurement errors on the classification performance of inspection policies. Further numerical analysis is conducted to examine the efficiencies of the different steps in a repeated inspection. The objective function and constraints are the same as those in the above numerical example. The obtained probabilities are listed in Table 3. It is obvious that the probability that a container is subjected to repeat inspection is small in all cases. This implies that majority of the unacceptable containers are detected in the first step of Policies II and III. Additional inspections are necessary for only a small portion of containers. This validates that repeat inspection under re-inspection band as described in Policy II and III does indeed detect most, if not all, the unacceptable containers. Note that a more precise sensor is typically more expensive with a higher cost per measurement. The fact that only a small portion of containers need to be re-inspected underlines an advantage of the proposed re-inspection policies, comparing to a single step non-repeated inspection with only the more precise sensor.

Table 3 Probabilities of containers subject to repeat inspections for Policy II and III

PFR / *PFA* <	Policy II	Policy III	
	Repeat Inspection	First Re-inspection	Second Re-inspection
0.001	0.0299	0.0432	0.0321
0.0001	0.0433	0.0736	0.058
0.00001	0.0577	0.0998	0.0802

Numerical examples presented in this section demonstrate that strategies such as the use of re-inspection band, repeat inspection, and repeat inspection by sensors with increased precision level indeed can reduce container false classification errors. Policy III which integrates all the proposed strategies attains the maximum improvement in terms of *PFR* relative to Policy I and II.

5 Modeling of container inspection system

At port-of-entry, containers arriving for inspection are either inherently acceptable or contain unacceptable materials, and they have several attributes which may reflect the presence or absence of such material. If we assume one sensor inspects one specific attribute and returns an acceptance-or-rejection decision (0 or 1 respectively), the container inspection problem can then be viewed from a system level. Since each sensor is subject to measurement errors, the modeling of single sensor inspection process considering measurement errors can be extended to the system level that considers all the attributes of the container.

An inspection decision system is expected to collect decisions from individual sensors (for a specific attribute of the container) and classify a container based on a system decision function *F* that assigns to each binary string of decisions $\left(D_1, D_2, \ldots, D_m\right)$ a category: 0 or 1 (accept or reject respectively). Elsayed *el al*. (2008) defined series, parallel, series-parallel, and parallel-series Boolean decision functions for an inspection system. All these Boolean decision functions can be applied in this work to integrate the decision from individual inspection station into the final decision regarding the acceptance of the container. For illustration purpose, in this paper, we only present the extension from a single station to an inspection decision system based on a series Boolean decision function. That is, all the container attributes are inspected in sequential inspection stations where each station checks the presence of one attribute and returns a decision D_i. The Boolean function *F* assigns a container class “1” if any of the individual decisions is fail, i.e. $D_i = 1$ for any sensor *i*. The Boolean function *F* assigns a container class “0” if all the individual decisions are acceptance, i.e. $D_i = 0$ for all sensor *i*. Since various risks may be indicated by different attributes, such a system decision function can warrant the rejection of a container upon detection any of the critical attributes.

Let $\left(x_1\ x_2 \cdots x_m\right)$ represent independent container attributes, such that $x_i = 0$ indicates the absence of attribute *i*, and $x_i = 1$ the presence of attribute *i*. In the overall container population the probability of presence of attribute *i* is $P(x_i = 1) = \pi_i$ and $P(x_i = 0) = 1 - \pi_i$. The true measurement of each attribute depends on the true status of the attribute (absence or presence), thus we assume two different distributions: $\left(y_i \mid x_i = 0\right) \sim N\left(\mu_{0i}, \sigma_{0,i}{}^2\right)$ and $\left(y_i \mid x_i = 1\right) \sim N\left(\mu_{1i}, \sigma_{1,i}{}^2\right)$. The random measurement error of each sensor is assumed to be normally distributed as $\varepsilon_i \sim N\left(0, \sigma_{\varepsilon,i}{}^2\right)$. The error term is independent of x_i. By the additive error model, $r_i = y_i + \varepsilon_i$, the observed readings from each sensor can be written as $\left(r_i \mid x_i = 0\right) \sim N\left(\mu_{0i}, \sigma_{0,i}{}^2 + \sigma_{\varepsilon,i}{}^2\right)$ and $\left(r_i \mid x_i = 1\right) \sim N\left(\mu_{1i}, \sigma_{1,i}{}^2 + \sigma_{\varepsilon,i}{}^2\right)$.

If an individual sensor makes a decision about a particular attribute following the procedure of Policy I, then systematically, the probability of false rejection and false acceptance of a container are respectively obtained as

$$PFR = 1 - \prod_{i=1}^{m} P\left(D_i = 0 \mid x_i = 0\right) \quad (1)$$

$$PFA = P\left(D = 0 \mid X = 1\right) = \frac{P\left(D = 0, X = 1\right)}{P\left(X = 1\right)}$$

$$= \frac{\sum_{x_i \in \{0,1\},\ \sum_{i=1}^{m} x_i \geq 1} \left\{ P\left[\left(D_1, ..., D_m\right) = \left(0, ..., 0\right) \mid \left(X_1, ..., X_m\right) = \left(x_1, ..., x_m\right)\right] \cdot P\left[\left(X_1, ..., X_m\right) = \left(x_1, ..., x_m\right)\right] \right\}}{1 - P\left(X = 0\right)}$$

$$= \frac{\sum_{x_i \in \{0,1\},\ \sum_{i=1}^{m} x_i \geq 1} \left\{ \prod_{i=1}^{m} \left[P\left(D_i = 0 \mid X_i = x_i\right) \cdot P\left(X_i = x_i\right) \right] \right\}}{1 - \prod_{i=1}^{m} \left(1 - \pi_i\right)}, \quad (2)$$

where $P\left(X_i = x_i\right) = \begin{cases} \pi_i & if \quad x_i = 1 \\ 1 - \pi_i & if \quad x_i = 0 \end{cases}$ and

$$P\left(D_i = 0 \mid X_i = x_i\right) = \begin{cases} \Phi\left(\dfrac{T_i - \mu_0}{\sqrt{\sigma_{0,i}^2 + \sigma_{\varepsilon,i}^2}}\right) & if \quad x_i = 0 \\ \Phi\left(\dfrac{T_i - \mu_1}{\sqrt{\sigma_{1,i}^2 + \sigma_{\varepsilon,i}^2}}\right) & if \quad x_i = 1 \end{cases}.$$

The optimization problem of the system inspection policy can be formulated as minimize PFR subject to $PFA \leq FA^*$, where PFR and PFA are computed as in (1) and (2). The decision variables are the thresholds T_i for each sensor.

If an individual sensor in the system uses Policy II or Policy III, the system's PFR and PFA can again be computed by (1) and (2), except that the expression of the corresponding $P\left(D_i = 0 \mid X_i = x_i\right)$ is more complicated and is given in the Appendix. In the optimization problem related to Policy II or III, the decision variables are the thresholds T_i, as well the widths of the re-inspection bands b_i, for each of the sensors.

6 Conclusions

In this paper, we consider a problem often encountered in the practice of container inspection at port-of-entry where measurements of inspection devices (or sensors) have errors. We develop measurement error models and formulate and study three inspection policies and compare their performance in terms of container misclassifications. We show that ignoring measurement error results in a higher percentage of container misclassification.

The inspection policies studied in this paper are not inclusive and other policies that can be used to deal with measurement errors. Moreover, the optimization of inspection policies of minimizing the PFR under PFA constraints can be generalized by considering the total inspection cost. In addition, the present work can be extended to systems with different structure of decision trees, such as parallel, parallel-series, series-parallel or other Boolean functions.

Finally, it is noted that the inspection Policy II is in fact equivalent to a special kind of two-step inspection under Policy III. Note that the average of the $l-1$ re-inspection measurements $\overline{r}_{2-l}$ follows the distribution of (1). It would be the same as the measurement of the first re-inspection if the precision level of the senor in the re-inspection $\sigma_{\varepsilon 2}$ equals $\sigma_{\varepsilon} \big/ \sqrt{l-1}$. In the numerical example, the gain in efficiency between Policy II and Policy III is far less than those improvements of re-inspection policies over Policy I. This reinforces the conclusion that a later re-inspection step tends to contribute less when comparing with an earlier re-inspection step for sensors with the same precisions.

7 Appendix

If an individual sensor makes a decision on an attribute following Policy II, then

$$P\left(D_i = 0 \mid X_i = x_i\right) =$$

$$\begin{cases} \Phi\left(\dfrac{T_i - \dfrac{b_i}{2} - \mu_0}{\sqrt{\sigma_{0,i}^2 + \sigma_{\varepsilon,i}^2}}\right) + \displaystyle\int_{-\infty}^{\infty} \Phi\left(\dfrac{T_i - y_i}{\sigma_{\varepsilon,i} \big/ \sqrt{l-1}}\right) \left\{ \Phi\left(\dfrac{T_i + \dfrac{b_i}{2} - y_i}{\sigma_{\varepsilon,i}}\right) - \Phi\left(\dfrac{T_i - \dfrac{b_i}{2} - y_i}{\sigma_{\varepsilon,i}}\right) \right\} \dfrac{\phi\left(\dfrac{y_i - \mu_0}{\sigma_{0,i}}\right)}{\sigma_{0,i}} dy_i & \text{if } x_i = 0 \\ \Phi\left(\dfrac{T_i - \dfrac{b_i}{2} - \mu_1}{\sqrt{\sigma_{1,i}^2 + \sigma_{\varepsilon,i}^2}}\right) + \displaystyle\int_{-\infty}^{\infty} \Phi\left(\dfrac{T_i - y_i}{\sigma_{\varepsilon,i} \big/ \sqrt{l-1}}\right) \left\{ \Phi\left(\dfrac{T_i + \dfrac{b_i}{2} - y_i}{\sigma_{\varepsilon,i}}\right) - \Phi\left(\dfrac{T_i - \dfrac{b_i}{2} - y_i}{\sigma_{\varepsilon,i}}\right) \right\} \dfrac{\phi\left(\dfrac{y_i - \mu_1}{\sigma_{1,i}}\right)}{\sigma_{1,i}} dy_i & \text{if } x_i = 1 \end{cases}$$

If an individual sensor makes a decision on a specific attribute according to Policy III, then

$$P\left(D_i = 0 \mid X_i = x_i\right) =$$

$$\begin{cases} \Phi\left(\dfrac{T_i - \dfrac{b_{i,1}}{2} - \mu_0}{\sqrt{\sigma_{0,i}^2 + \sigma_{\varepsilon,1,i}^2}}\right) + \displaystyle\sum_{k=2}^{l-1} \int_{-\infty}^{\infty} \Phi\left(\dfrac{T_i - \dfrac{b_{i,k}}{2} - y_i}{\sigma_{\varepsilon,k,i}}\right) \prod_{j=1}^{k-1} \left\{ \Phi\left(\dfrac{T_i + \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) - \Phi\left(\dfrac{T_i - \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) \right\} \dfrac{\phi\left(\dfrac{y_i - \mu_0}{\sigma_{0,i}}\right)}{\sigma_{0,i}} dy_i \\ + \displaystyle\int_{-\infty}^{\infty} \Phi\left(\dfrac{T_i - y_i}{\sigma_{\varepsilon,l,i}}\right) \prod_{j=1}^{l-1} \left\{ \Phi\left(\dfrac{T_i + \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) - \Phi\left(\dfrac{T_i - \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) \right\} \dfrac{\phi\left(\dfrac{y_i - \mu_0}{\sigma_{0,i}}\right)}{\sigma_{0,i}} dy_i & \text{if } x_i = 0 \\ \Phi\left(\dfrac{T_i - \dfrac{b_{i,1}}{2} - \mu_1}{\sqrt{\sigma_{1,i}^2 + \sigma_{\varepsilon,1,i}^2}}\right) + \displaystyle\sum_{k=2}^{l-1} \int_{-\infty}^{\infty} \Phi\left(\dfrac{T_i - \dfrac{b_{i,k}}{2} - y_i}{\sigma_{\varepsilon,k,i}}\right) \prod_{j=1}^{k-1} \left\{ \Phi\left(\dfrac{T_i + \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) - \Phi\left(\dfrac{T_i - \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) \right\} \dfrac{\phi\left(\dfrac{y_i - \mu_1}{\sigma_{1,i}}\right)}{\sigma_{1,i}} dy_i \\ + \displaystyle\int_{-\infty}^{\infty} \Phi\left(\dfrac{T_i - y_i}{\sigma_{\varepsilon,l,i}}\right) \prod_{j=1}^{l-1} \left\{ \Phi\left(\dfrac{T_i + \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) - \Phi\left(\dfrac{T_i - \dfrac{b_{i,j}}{2} - y_i}{\sigma_{\varepsilon,j,i}}\right) \right\} \dfrac{\phi\left(\dfrac{y_i - \mu_1}{\sigma_{1,i}}\right)}{\sigma_{1,i}} dy_i & \text{if } x_i = 1 \end{cases}$$

Acknowledgements This research is conducted with partial support from Office of Naval Research, the National Science Foundation and the National Security Agency. The authors wish to thank Fred Roberts, Paul Kantor, and other members of the DIMACS Port-of-Entry research group for their comments and input throughout the research.

References

Boros, E., Fedzhora, L., Kantor, P. B., Saeger, K., & Stroud, P. (2008). Large scale LP model for finding optimal container inspection strategies. *Naval Research Logistics* in press.

Brandenstein, A. E. (2007). Article number 4373477. *Proceedings of International Carnahan Conference on Security Technology 2007*, 109-116.

Chandra, J. & Schall, S. (1988). The use of repeated measurements to reduce the effects of measurement errors. *IIE Transactions*, *20*(1), 83-87.
Chen, S.-L. & Chung, K.-J. (1996). Selection of the optimal precision level and target value for a production process: the lower-specification limit case. *IIE Transactions*, *28*, 979-958.
Dahlman, O., Mackby, J., Sitt, B., Poucet, A., Meerburg, A., Massinon, B., Ifft, E., Asada, M., & Alewine, R. (2005). Container security:A proposal for a comprehensive code of conduct. *Defense & Technology Papers published by the National Defense University Center for Technology and National Security Policy*, Fort Lesley J. McNair, Washington DC. Available: http://www.ndu.edu/ctnsp/publications.html.
Deaver, D. (1995). Using guardbands to justify TURs less than 4:1. *In: Proceeding of ASQC 49th Annual Quality Congress*, 136-141.
Dye, D. H., Sensors for screening and surveillance (2003). *Paper presented at the Hoover Institute National Security Forum Conference on Technology for Preventing Terrorism, CA*.
Eagle, A. R. (1954). A method for handling errors in testing and measuring. *Industrial Quality Control*, *10*, 10-15.
Elsayed, E. A., Young, C. M., Xie, M., Zhang, H., & Zhu, Y. (2008). Port-of-entry inspection: sensor deployment policy optimization. Accepted by *IEEE Transactions on Automation Science and Engineering.*
Grubbs, F. A. & Coon, H. J. (1954). On setting test limits relative to specification limits. *Industrial Quality Control*, *10*, 15-20.
Hutchinson, B. (1991). Setting guardband test limits to satisfy MIL-STD-45662A requirements. In: Proceeding of 1991 NCSL Workshop and Symposium, 305-309.
Kim, Y. J., Cho, B. R., & Kim, N. (2007). Economic design of inspection procedures using guard band when measurement errors are present. *Applied Mathematical Modeling*, *31*, 805-816.
Koch, D. B. (2007). PortSim-A port security simulation and visualization tool. *Security Technology 2007 41st Annual IEEE International Carnahan Conference on 8-11 October 2007*, 109-116.
Mader, D. P., Prins, J., Lampe., R. E. (1999). The economic impact of measurement error. *Quality Engineering*, *11*, 563-574.
McCarville, D. R. & Montgomery, D. C. (1996). Optimal guard bands for gauges in series. *Quality Technology*, *9*, 167-177.
Ramirez-Marquez, J. E. (2008). Port-of-entry safety via the reliability optimization of container inspection strategy through an evolutionary approach. *Reliability Engineering and System Safety*, *93*, 1698-1709.
Stroud, P. D., & Saeger, K. J. (2003). Enumeration of increasing Boolean expressions and alternative digraph implementations for diagnostic applications. *In: Proceedings Volume IV, Computer, Communication and Control Technologies*: I, (eds. Chu, H., Ferrer, J., Nguyen, T., & Yu, Y.), 328-333.
Tang, K. & Schneider, H. (1988). Selection of the optimal inspection precision level for a complete inspection plan. *Journal of Quality Technology*, *20*, 153-156.
Wein, L. M., Wilkins,A. H., Baveja, M., & Flynn, S. E. (2006). Preventing the Importation of Illicit Nuclear Materials in Shipping Containers. *Risk Analysis*, *26*(5), 1377-1393.