Sample questions

Question 3:

a) The roots to the characteristic polynomial are $.3 \pm .56i$. The roots are imaginary and paired so we expect the acf to look periodic.
b) The roots are within the unit circle so the process is causal.
c) For $m = 1$ it's easy to see that the predictor takes the same form as the AR model: $E[y_{t+1}|y_1^t] = .6y_t - .4y_{t-1}$. The MSE is σ^2 for the one-step predictor.
For $m = 3$ we compute $E[y_{t+3}|y_1^t]$ iteratively (keep on iterating the AR model formulation until you have only a function of observed values) and finally obtain $-.264y_t + .016y_{t-1}$. The MSE is $E[e_{t+3} + .6e_{t+2} - .04e_{t+1}]^2 = 1.3616\sigma^2$.
d) As m grows the MSE approaches the marginal variance of y_t.

Question 4:

a) We will use the relationship $f_x(\lambda) = |A(\lambda)|^2 f_s(\lambda) + f_e(\lambda)$. $A(\lambda) = 1 + ae^{-i\lambda D}$, such that $|A(\lambda)|^2 = (1 + a^2 + 2a\cos(\lambda D))$. Done.
b) If $s_t = s_{t-D} + \eta_t$ and $a = -1$, D will be "lost" in x_t. If s_t has a smoothly varying f_s and $a \neq -1$ we expect to see a peak in the spectrum f_x at $\lambda = 2\pi k/D$, and 0 when $\cos(\lambda D) = -(1 + a^2)/2a$. We can solve for these λ and check.
c) If the input is WN we expect the acf to exhibit the periodicity associated with D.