There were a couple of emails regarding question 3. So, I am including solution to problem 3. The rest are straightforward. You can mail me at sasi@stat.rutgers.edu if you have any trouble.

Introductory Statistics-I

Sample Final Exam

Answer all the questions. The maximum you can score is 100 points. Clearly state your assumptions and any results/facts you use.

(1)A study was conducted to determine if elite class distance runners are actually thinner than other people. The researchers measured skin fold thickness, an indirect indicator of body fat. The sample data, in millimeters(mm), is as follows:

RUNNERS: 7.3,3.0,7.8,5.4,3.7,6.7,5.1,3.8,6.4,7.5,8.7,8.8,6.2,6.3,4.6.
OTHERS: 24.0,28.0,9.3,9.6,12.4,19.9,29.4,18.1,19.4,5.2,7.5,20.3,22.8,16.3,12.2.

a) Find the mean and median of the skin fold thickness for both the groups. [5]
b) Use boxplots to compare these two data sets, paying special attention to center and variation. [10]

(2) According to an opinion poll, 54% of U.S. men and 33% of U.S. women believe in aliens. Of U.S. adults, 48% are men and 52% are women.
a) What percentage of U.S. adults believe in aliens? [5]
Ans: 0.4308
b) What percentage of U.S. women believe in aliens? [5]
Ans: 0.33
c) What percentage of U.S. adults that believe in aliens are women? [5]
Ans: 0.3983

(3)In a horse race, the probability that the frontrunner horse Ace wins the race is 0.65. In the second week of December, five races are to be conducted and a gambler is required to bet on the same horse for all the five races. A gambler wins $500 for every win and also a grand prize of $1000 if the horse he/she bets on, wins more than 2 races. For example, if Ace wins 3 races, any gambler betting on Ace would receive $2500.
a) If Lucky Joe bets on Ace, what is the probability that he wins the grand prize? [5]

Solution: Let X be defined to be defined as the number of races Ace wins. Clearly , X is a Binomial random variable with n=5 and probability of success p=0.65. Lucky Joe gets the grand prize if Ace wins more than two races. So, P(Ace wins more than 2 races)=$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) = 0.7648$.

b) Define Y to be the prize money Lucky Joe won, by betting on Ace. Find the mean value of the random variable Y. [10]

Solution: There are a couple of ways to do this problem.

Method 1: Let Y be defined as the amount of money Lucky Joe wins betting on Ace. The following gives the probability distribution of Y. Y depends on how many races Ace wins. Observe the following.

$X = 0 \Leftrightarrow Y = 0$

$X = 1 \Leftrightarrow Y = 500$

$X = 2 \Leftrightarrow Y = 1000$

$X = 3 \Leftrightarrow Y = 2500$

$X = 4 \Leftrightarrow Y = 3000$

$X = 5 \Leftrightarrow Y = 3500$ where $\Leftrightarrow$ stands for 'if and only if'.

So, by definition, mean value of Y is $\mu_Y = \sum yP(Y = y)$

$= 0.P(Y = 0)+500.P(Y = 500)+1000.P(Y = 1000)+2500.P(Y = 2500)+3000.P(Y = 3000)+3500.P(Y = 3500)$

$= 0 + 500.P(X = 1) + 1000.P(X = 2) + 2500.P(X = 3) + 3000.P(X = 4) + 3500.P(X = 5)$

$= 500.\binom{5}{1}.0.65^1.0.35^4+1000.\binom{5}{2}.0.65^2.0.35^3+2500.\binom{5}{3}.0.65^3.0.35^2+3000.\binom{5}{4}.0.65^4.0.35^1+3500.\binom{5}{5}.0.65^5.0.35^0$

=2389.80

Method 2: The prize money Y can be divided into two parts Y_1 and Y_2. Y_1 consists of the prize money without considering the grand prize, ie, $Y_1 = 500X$. Y_2 consists of the grand prize part. So, Y_2 is either zero or \$1000, depending on the number of times Ace wins the race.

$Y_2 = 0$ if $X \leq 2$

$Y_2 = 1000$ if $X > 2$

$\mu_Y = \mu_{Y_1} + \mu_{Y_2} = 500\mu_X + \mu_{Y_2}$

Notice that X is a Binomial variable and Y_2 is a Bernoulli variable. Hence,

$\mu_Y = 500\mu_X + \mu_{Y_2}$=500.(5).(0.65)+1000.(0.7648)=2389.80

(4) The times of the finishers in the New York 10km run are normally distributed with mean 61 min-

utes and standard deviation 9 minutes.

a) Determine the percentage of finishers with times between 52 and 70 minutes. [5]

ans: 0.6826

b) Find the 60^{th} percentile of the finishing times. [5]

Ans: 63.295

c) A team of four members is said to finish in time, if their average finishing time is less than one hour. What is the probability that a given team of four finishes in time? [5]

Ans: 0.4129

(5) For 36 randomly selected Rolling Stones concerts, the mean gross earnings is \$2.27 million.

a) Assuming a population standard deviation of \$0.5 million, obtain a 99% confidence interval for the mean earnings of Rolling Stones concerts. [10]

Ans: (2.184, 2.355) in millions of dollars

b) What is the minimum sample size needed in order to be 95% confident that the population mean earnings μ is within \$100,000 of the sample mean. [5]

Ans: 97

c) Assume now that the population standard deviation is unknown and that the sample of 36 concerts has a sample standard deviation of \$0.6 million. Find a 95% confidence interval for the population mean earnings μ. [10]

Ans: (2.067,2.473) in millions of dollars

(6) In the year 2000, the mean retail price of agriculture books was \$66.52. This year's retail prices for 28 randomly selected agriculture books are shown below.

68.45 76.61 66.01 55.02 55.77 71.78 75.31

60.99 46.58 59.38 69.33 47.05 67.12 56.26

59.36 79.94 57.05 75.09 68.27 50.54 62.57

58.86 67.99 66.95 55.56 75.67 59.52 75.59

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a) At the 10% level of significance, do the data provide sufficient evidence to conclude that this year's mean retail price of agriculture books has changed from the 2000 mean? Assume that the standard deviation of prices for this year's agriculture books is \$8.45. (Note: The sum of the data is \$1788.62) [15]

Ans:

$H_0 : \mu = 66.52$

$H_0 : \mu \neq 66.52$

Test Statistic $Z = \frac{\sqrt{n}(\overline{X}-\mu)}{\sigma}$

Rejection region: Reject if $Z > 1.645$ or if $Z < -1.645$

Value of the test statistic $Z = \frac{\sqrt{n}(\overline{X}-\mu)}{\sigma} = -1.653704$

Hypothesis test decision: Reject H_0.

GOOD LUCK