

Theorem of black and white balls

In the lecture on Sept 14, we obtained the following result, which is so important that I call it “Theorem of black and white balls” for emphasis and for ease of reference. Keep this result in mind and you will find how useful it is to solve problems in probability.

Theorem of black and white balls. A box contains a total of N balls, in which there are M black balls and $N - M$ white balls.

1) Randomly draw n balls out of the box without replacement. The probability that m black balls and $n - m$ white balls are obtained is

$$\frac{\binom{M}{m} \binom{N-M}{n-m}}{\binom{N}{n}}.$$

2) Randomly draw n balls out of the box with replacement. The probability that m black balls and $n - m$ white balls are obtained is

$$\binom{n}{m} \left(\frac{M}{N}\right)^m \left(1 - \frac{M}{N}\right)^{n-m}.$$