

Statistics 381

PRACTICE EXAM TO MIDTERM #1

1 hour and 20 minutes

1. [60 points] Two boxes are labelled I and II. Box I contains 4 black balls and 6 white balls. Box II contains 2 black balls and 8 white balls. A player randomly draws one ball from each box (so that a total of 2 balls are obtained).

1) What is the probability of obtaining at least one black ball?

2) What is the probability of obtaining exactly one black ball?

3) What is the probability of obtaining no black ball?

4) What is the probability of obtaining 2 black balls?

5) Given that at least one black ball is obtained, what is the probability that exactly one black ball is obtained?

6) Given that at least one black ball is obtained, what is the probability that the ball from box I is black?

7) Given that exactly one black ball is obtained, what is the probability that the ball from box I is black?

Let X be the number of black balls obtained by the player.

8) Find the probability mass function of X .

Suppose that the player is awarded \$3 for obtaining a black ball and \$1 for obtaining a white ball. Let Y be the total amount of dollars awarded to the player.

9) Find the probability mass function of Y .

10) Find the mean and the variance of Y .

Suppose that the player returns the 2 balls to their original boxes respectively, and repeats the game of drawing 1 ball from each box. Let Z be the number of times the game is played until the player obtains at least 1 black ball.

11) Find the probability mass function of Z .

12) Find the mean and the variance of Z .

2. [20 points] Two boxes are labelled I and II. Box I contains 4 black balls and 6 white balls. Box II contains 2 black balls and 8 white balls. A player randomly selects one box and then randomly draws 2 balls without replacement from the selected box.

1) What is the probability of obtaining 2 black balls?

2) Given that 2 black balls are obtained, what is the probability that Box I is selected?

Suppose that 20 players conduct the game independently.

3) What is the probability that exactly 4 players obtain 2 black balls?

4) Find the Poisson approximation to your answer in 3).

3. [20 points] A box contains 2 black balls and 3 white balls. A player randomly draws balls from the box one by one, without replacement. Let X be the number of draws until both black balls are obtained.

1) Find $P(X = 3)$.

2) Find the probability mass function of X .

3) Find the mean and the variance of X .

4) Find and graph the cumulative distribution function of X .