Solutions to HW 1, even-numbered and extra problems

Note: The solutions to the odd-numbers problems can be found in *Student Solutions Manual* to Wackerly, Mendenhall, and Scheaffer (2002).

2.8

a. The sample space consists of the four possible blood phenotypes. That is,

$$S = \{A, B, AB, O\}.$$

b. The probabilities may be assigned as follows.

$$P(\{A\}) = 0.41,\ P(\{B\}) = 0.10,\ P(\{AB\}) = 0.04,\ P(\{O\}) = 0.45.$$

c. Since the events $\{A\}$ and $\{B\}$ are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B) = 0.41 + 0.10 = 0.51.$$

2.12 Extra problems

1) The simple events are

{needs glasses and uses them}, {needs glasses and does not use them}
{does not need glasses and uses them}, {does not need glasses and does not use them}

2) The simple events are not equally likely. We should assign probabilities based on the given proportions. That is,

$$P(\text{needs glasses and uses them}) = 0.44$$
$$P(\text{needs glasses and does not use them}) = 0.14$$
$$P(\text{does not need glasses and uses them}) = 0.02$$
$$P(\text{does not need glasses and does not use them}) = 0.40$$

2.12

a. $P(\text{ needs glasses }) = 0.44 + 0.14 = 0.58$

b. $P(\text{ needs glasses and does not use them }) = 0.14$

c. $P(\text{ uses glasses }) = 0.44 + 0.02 = 0.46$

2.20 Denote the four candidates as A_1, A_2, A_3, and M, where M is the member of a minority group.

a. & b. Order is unimportant in the pair chosen. Hence there are six possible outcomes, each with probability 1/6.

$$E_1 : \{A_1 A_2\} E_2 : \{A_1 A_3\} \qquad E_3 : \{A_1 M\}$$
$$E_4 : \{A_2 A_3\} E_5 : \{A_3 M\} \qquad E_6 : \{A_3 M\}$$

c. $P(\text{minority hired}) = P(E_3) + P(E_5) + P(E_6) = 3/6 = 1/2$.

2.22

a. Let $w1$ denote the first wine, $w2$ the second, and $w3$ the third. Then one sample point would be an ordered triple indicating the rank of each wine. For example, $(w1, w2, w3)$ would indicate that $w1$ is superior to $w2$ and $w3$ while $w2$ is superior to just $w3$.

b. The sample space is given by all the possible ordered triples. That is,

$$(w1, w2, w3),\ (w1, w3, w2),\ (w2, w1, w3),$$
$$(w2, w3, w1),\ (w3, w1, w2),\ (w3, w2, w1)$$

c. Suppose $w1$ is superior to $w2$ and $w3$. Then the probability that the "expert" ranks $w1$ as first or second is

$$P((w1, w2, w3) \text{ or } (w1, w3, w2) \text{ or } (w2, w1, w3) \text{ or } (w3, w1, w2)) = 4/6 = 2/3.$$

2.36 Recall from Question **2.35**, the number of all possible outcomes is

$$\binom{9}{3\ 5\ 1} = \binom{9}{3}\binom{6}{5}\binom{1}{1} = 504.$$

a. The broken taxi is dispatched to airport C. Among the 8 unbroken taxi, 3 of them are dispatched to airport A and the remaining 5 are dispatched to airport B. The number of ways of doing this is

$$\binom{8}{3\ 5} = \binom{8}{3}\binom{5}{5} = 56.$$

The probability is $56/504 = 1/9$.

b. There are 3 broken taxi and 6 unbroken taxi. Among the 3 broken taxi, 1 of them is dispatched to airport A, 1 of them to airport B, and 1 of them to airport C. Among the

6 unbroken airport, 2 of them are dispatched to airport A and 5 of them to airport B. The number of ways of doing this is

$$\binom{3}{1\ 1\ 1} \times \binom{6}{2\ 4} = \binom{3}{1}\binom{2}{1}\binom{1}{1} \times \binom{6}{2}\binom{4}{4} = 90.$$

The probability is 90/504.

2.50 The number of all possible outcomes is 6^6. The number of ways in which the numbers recorded are 1, 2, 3, 4, 5, 6 in any order is 6!. The probability is

$$\frac{6!}{6^6} = \frac{5}{324}$$

2.52

a. After assigning an ethnic group member to each type of job, there are16 laborers remaining to assign to the remaining jobs. Let n_a be the number of ways that one ethnic group member can be assigned to each type of job. Then

$$n_a = \binom{4}{1\ 1\ 1\ 1}\binom{16}{5\ 3\ 4\ 4}$$

and hence the probability is $n_a/N = .1238$

b. Let n_a be the number of ways that no ethnic member gets assigned to a type 4 job, that is, all 4 ethnic members get assigned to type 1, 2, or 3 job. Consider type 1, 2, and 3 together as a single "merged" type, which requires $6 + 4 + 5 = 16$ laborers. Then

$$n_a = \binom{4}{4\ 0}\binom{16}{11\ 5}$$

and the desired probability is

$$\frac{\binom{4}{0}\binom{16}{5}}{\binom{20}{5}} = .2817$$