Solutions to HW 2, even-numbered and extra problems

Note: The solutions to the odd-numbers problems can be found in *Student Solutions Manual* to Wackerly, Mendenhall, and Scheaffer (2002).

2.60 Define the events

$$U : \text{job is unsatisfactory} \quad \text{and} \quad A : \text{plumber A does the job.}$$

It is given that $P(A) = .4$, $P(U) = .1$, and $P(A|U) = .5$.

a. The probability is

$$P(U|A) = \frac{P(A \cap U)}{P(A)} = \frac{P(A|U)P(U)}{P(A)} = \frac{(.5)(.1)}{.4} = .125.$$

b. $P(\overline{U}|A) = 1 - P(U|A) = 1 - .125 = .875.$

2.61 Extra problems

$$\begin{aligned} P(A|A \cup B) &= \frac{P(A)}{P(A \cup B)} = \frac{.40}{.67}, \\ P(A|A \cap B) &= 1, \\ P(A \cup B|A \cap B) &= 1, \\ P(A \cap B|A \cup B) &= \frac{P(A \cap B)}{P(A \cup B)} = \frac{.10}{.67}. \end{aligned}$$

2.72 Note that $P(A \cap B) = P(A)P(B)$ because A and B are independent.

a. $P(A \cup B) = P(A) + P(B) - P(A)P(B) = .5 + .2 - (.5)(.2) = .6.$

b. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - .6 = .4.$

c. $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B) = 1 - .1 = .9.$

2.78 Define the events

$$I : \text{disease I is contracted} \quad \text{and} \quad II : \text{disease II is contracted.}$$

Then $P(I) = .1$, $P(II) = .15$, and $P(I \cap II) = .03$.

a. $P(I \cup II) = P(I) + P(II) - P(I \cap II) = .1 + .15 - .03 = .22.$

b. $P(I \cap II|I \cup II) = \frac{P(I \cap II)}{P(I \cup II)} = .03/.22.$

2.90 Define the events

$$T: \text{positive reading for the truthful person} \quad \text{and} \quad L: \text{positive reading for the liar.}$$

Then $P(T) = .1$, $P(L) = .95$, and T and L are independent.

a. $P(T \cap L) = P(T)P(L) = .1 \times .95 = .095.$

b. $P(\overline{T} \cap L) = P(\overline{T})P(L) = (1 - .1) \times .95 = .855.$

c. $P(T \cap \overline{L}) = P(T)P(\overline{L}) = .1 \times (1 - .95) = .005.$

d. $P(T \cup L) = 1 - P(\overline{T} \cap \overline{L}) = 1 - P(\overline{T})P(\overline{L}) = 1 - (1 - .1) \times (1 - .95) = .955.$

2.90 Extra problems

1) The event that the detector gives a positive reading and a negative reading is $(T\cap\overline{L})\cup(\overline{T}\cap L)$. The events $T\cap\overline{L}$ and $\overline{T}\cap L$ are mutually exclusive. So $P\{(T\cap\overline{L})\cup(\overline{T}\cap L)\} = P\{T \cap \overline{L}\} + P\{\overline{T} \cap L\} = .005 + .855 = .86$. The conditional probability is

$$P\{T \cap \overline{L} | (T \cap \overline{L}) \cup (\overline{T} \cap L)\} = \frac{P\{T \cap \overline{L}\}}{P\{(T \cap \overline{L}) \cup (\overline{T} \cap L)\}} = \frac{.005}{.86} = .0058.$$

2) Note that $(T \cap \overline{L}) \subset (T \cup L)$. The conditional probability is

$$P\{T \cap \overline{L} | T \cup L\} = \frac{P\{T \cap \overline{L}\}}{P\{T \cup L\}} = \frac{.005}{.955} = .0052.$$

2.96 Consider the 6 refrigerators as 6 balls, the 2 defective ones as 2 black balls, and the 4 non-defective ones as 4 white balls.

a. In class, we showed that the probability is 1/5.

b. The event that no more than 4 balls need to be drawn to obtain both black balls is equivalent to saying that the first 4 draws contain 2 black balls. By Theorem of black-and-white-balls, the probability is

$$\frac{\binom{2}{2}\binom{4}{2}}{\binom{6}{4}} = \frac{2}{5}.$$

c. Given that exactly one of the 2 black balls is obtained in the first 2 draws, there are 1 black ball and 3 white balls left, and hence the probability that the remaining 1 black ball is obtained in the next 2 draws is (again by Theorem of black-and-white-balls)

$$\frac{\binom{1}{1}\binom{3}{1}}{\binom{4}{2}} = \frac{2}{5} = \frac{1}{2}.$$

2.100 Define the events

$$R: \text{the specimen turns red} \quad \text{and} \quad N: \text{the specimen contains nitrate.}$$

a. By the law of total probability

$$\begin{aligned} P(R) &= P(R|N)P(N) + P(R|\overline{N})P(\overline{N}) \\ &= (.95)(.3) + (.1)(1-.3) = .355. \end{aligned}$$

b. BY Bayes's rule,

$$P(N|R) = \frac{P(R|N)P(N)}{P(R)} = \frac{(.95)(.3)}{.355} = .803.$$

2.104 Define the events

$$C: \text{contract lung cancer} \quad \text{and} \quad N: \text{work in a shipyard.}$$

By Bayes's rule,

$$\begin{aligned} P(C|S) &= \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|\overline{C})P(\overline{C})} \\ &= \frac{(.22)(.0004)}{(.22)(.0004) + (.14)(1-.0004)} = .0006. \end{aligned}$$