Solutions to HW 3, even-numbered and extra problems

Note: The solutions to the odd-numbers problems can be found in *Student Solutions Manual* to Wackerly, Mendenhall, and Scheaffer (2002).

3.10

$$
\begin{aligned}
E(Y) &= \sum yp(y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2.0 \\
E(1/Y) &= \sum (1/y)p(y) = 1(.4) + (1/2)(.3) + (1/3)(.2) + (1/4)(.1) = .6417 \\
E(Y^2 - 1) &= E(Y^2) - 1 = [1(.4) + 4(.3) + 9(.2) + 16(.1)] - 1 = 5 - 1 = 4 \\
V(Y) &= E(Y^2) - E^2(Y) = 5 - 2^2 = 1
\end{aligned}
$$

3.20 Let Y be daily sales. Then Y can take on three possible values, 0, 50000, or 100000. Note

$$
\begin{aligned}
P(Y = 0) &= P(\text{contact one, fail to sell}) + P(\text{contact two, fail to sell}) \\
&= P(\text{contact one})P(\text{fail to sell}) + P(\text{contact two})P(\text{fail with 1st})P(\text{fail with 2nd}) \\
&= (1/3)(9/10) + (2/3)(9/10)(9/10) = 252/300 \\
P(Y = 50000) &= P(\text{contact one, sell}) + P(\text{contact two, sell to one}) \\
&= P(\text{contact one, sell}) + P(\text{contact two, sell to 1st only}) \\
&\qquad + P(\text{contact two, sell to 2nd only}) \\
&= (1/3)(1/10) + (2/3)(1/10)(9/10) + (2/3)(9/10)(1/10) = 46/300 \\
P(Y = 100000) &= P(\text{contact two, sell to both}) \\
&= (2/3)(1/10)(1/10) = 2/300
\end{aligned}
$$

Then

$$
\begin{aligned}
E(Y) &= \sum yP(Y = y) \\
&= 0(252/300) + 50000(46/300) + 100000(2/300) = 25000/3
\end{aligned}
$$

3.42 The random variable Y is binomial with $n = 4$, $p = .1$. Therefore

$$
\begin{aligned}
E(Y) &= np = .4 \\
E(Y^2) &= V(Y) + E^2(Y) = npq + n^2p^2 = 4(.1)(.9) + (.4)^2 = .52
\end{aligned}
$$

Then $E(C) = 3E(Y^2) + E(Y) + 2 = 3(.52) + .4 + 2 = 3.96$.

3.58

$$\mu = \frac{1}{.9} = 1.11$$
$$\sigma = \sqrt{\frac{1-.9}{(.9)^2}} = \sqrt{.123} = .35$$

3.62 Let $Y =\#$ of toss on which first 6 appears. Then Y is geometric with $p = 1/6$.

$$\begin{aligned} P(\text{B tosses first 6}) &= P(Y = 2, 4, 6, ...) \\ &= p(2) + p(4) + p(6) + \cdots \\ &= \sum_{i=1}^{\infty} p(2i) = \sum_{i=1}^{\infty} q^{2i-1}p = \frac{p}{q}\sum_{i=1}^{\infty} q^{2i} \\ &= \frac{p}{q}\left(\frac{1}{1-q^2} - 1\right) = \frac{pq}{1-q^2} \end{aligned}$$

Sine $p = 1/6$, $P(\text{B tosses first 6}) = 5/11$. Therefore

$$P(Y = 4|\text{B tosses first 6}) = \frac{P(Y=4)}{P(\text{B tosses first 6})} = \frac{(\frac{5}{6})^3\frac{1}{6}}{\frac{5}{11}} = \frac{275}{1296}.$$

3.76 a.

$$\mu = \frac{1}{p} = \frac{1}{.9} = 1.11$$
$$\sigma^2 = \frac{1-.9}{(.9)^2} = \frac{.1}{.81} = .123$$

b.

$$\mu = \frac{r}{p} = \frac{3}{.9} = 3.33$$
$$\sigma^2 = \frac{3(1-.9)}{(.9)^2} = \frac{.3}{.81} = .3704$$

3.78 a. Let $Y =\#$ of attempts until you complete your call. Then Y is geometric with $p = .4$.

$$P(\text{complete the call on the first try}) = P(Y = 1) = .4$$
$$P(\text{complete the call on the second try}) = P(Y = 2) = (.4)(.6) = .24$$
$$P(\text{complete the call on the third try}) = P(Y = 3) = (.4)(.6)^2 = .144$$

b. Let $Y =\#$ of attempts until both calls are completed. Then Y is negative binomial with $r = 2$, $p = .4$.

$$P(Y = 4) = \binom{3}{1}(.4)^2(.6)^2 = .1728$$