

Problem 1. Note that

$$P(\text{ pass the theoretical section }) = .8$$

$$P(\text{ pass the applied section }) = .7$$

$$P(\text{ pass both sections }) = .6$$

1) $P(A \cap B) = P(\text{ pass both sections }) = .6$, which differs from $P(A)P(B) = (.8)(.7) = .56$. Therefore, A and B are not independent.

2)

$$\begin{aligned} P(\text{ pass at least one section }) &= P(A \cup B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= .8 + .7 - .6 = .9 \end{aligned}$$

3)

$$\begin{aligned} P(\text{ pass the applied but fail the theoretical }) &= P(\bar{A} \cap B) \\ &= P(B) - P(A \cap B) \\ &= .7 - .6 = .1 \end{aligned}$$

4)

$$\begin{aligned} P(\text{ pass the theoretical but fail the applied }) &= P(A \cap \bar{B}) \\ &= P(A) - P(A \cap B) \\ &= .8 - .6 = .2 \end{aligned}$$

Therefore,

$$\begin{aligned} &P(\text{ pass exactly one section }) \\ &= P(\bar{A} \cap B) + P(A \cap \bar{B}) \\ &= .1 + .2 = .3 \end{aligned}$$

5)

$$\begin{aligned} P(\text{ pass neither section }) &= P(\bar{A} \cap \bar{B}) \\ &= 1 - P(A \cup B) \\ &= 1 - .9 = .1 \end{aligned}$$

6)

$$\begin{aligned} & P(\text{ pass both sections } | \text{ pass at least one section }) \\ &= \frac{P(\text{ pass both sections })}{P(\text{ pass at least one section })} \\ &= \frac{.6}{.9} = \frac{2}{3} \end{aligned}$$

7)

$$\begin{aligned} & P(\text{ pass the applied section } | \text{ pass at least one section }) \\ &= \frac{P(\text{ pass the applied section })}{P(\text{ pass at least one section })} \\ &= \frac{.7}{.9} = \frac{7}{9} \end{aligned}$$

8)

$$\begin{aligned} & P(\text{ pass the applied section } | \text{ pass exactly one section }) \\ &= \frac{P(\text{ pass the applied but fail the theoretical})}{P(\text{ pass exactly one section })} \\ &= \frac{.1}{.3} = \frac{1}{3} \end{aligned}$$

Problem 2.

1) Let $A = \{ 2 \text{ black balls } \}$ and $B = \{ \text{ box I is selected } \}$. Then

$$\begin{aligned} P(A) &= P(A|B)P(B) + P(A|\bar{B})P(\bar{B}) \\ &= \left(\frac{4}{10}\right)^2 \frac{1}{2} + \left(\frac{2}{10}\right)^2 \frac{1}{2} \\ &= \frac{4}{25} \frac{1}{2} + \frac{1}{25} \frac{1}{2} = \frac{1}{10} \end{aligned}$$

2)

$$\begin{aligned} P(B|A) &= \frac{P(A|B)P(B)}{P(A)} \\ &= \frac{2/25}{1/10} = \frac{4}{5} \end{aligned}$$

3)

$$\binom{100}{6} \left(\frac{1}{10}\right)^6 \left(1 - \frac{1}{10}\right)^{94} = .0596$$

4)

$$\begin{aligned} & \binom{100}{6} \left(\frac{1}{10}\right)^6 \left(1 - \frac{1}{10}\right)^{94} \\ & \approx \frac{e^{-10} 10^6}{6!} = .0631 \end{aligned}$$

where $\lambda = np = 100(1/10) = 10$.

Problem 3.

1) See the answer to 2).

2) For $x = 1, 2, 3$,

$$\begin{aligned} P(X = x) &= P\{\text{select the brand ranked } x \text{ and two brands ranked higher than } x\} \\ &= \frac{\binom{5-x}{2}}{\binom{5}{3}} \end{aligned}$$

because there are $\binom{5}{3}$ possible ways to select three brands out of five brands, and there are $\binom{5-x}{2}$ possible ways to select two brands out of $5-x$ brands ranked higher than x , in addition to selecting the brand ranked x . That is,

$$P(X = 1) = \frac{\binom{4}{2}}{\binom{5}{3}} = \frac{6}{10}$$

$$P(X = 2) = \frac{\binom{3}{2}}{\binom{5}{3}} = \frac{3}{10}$$

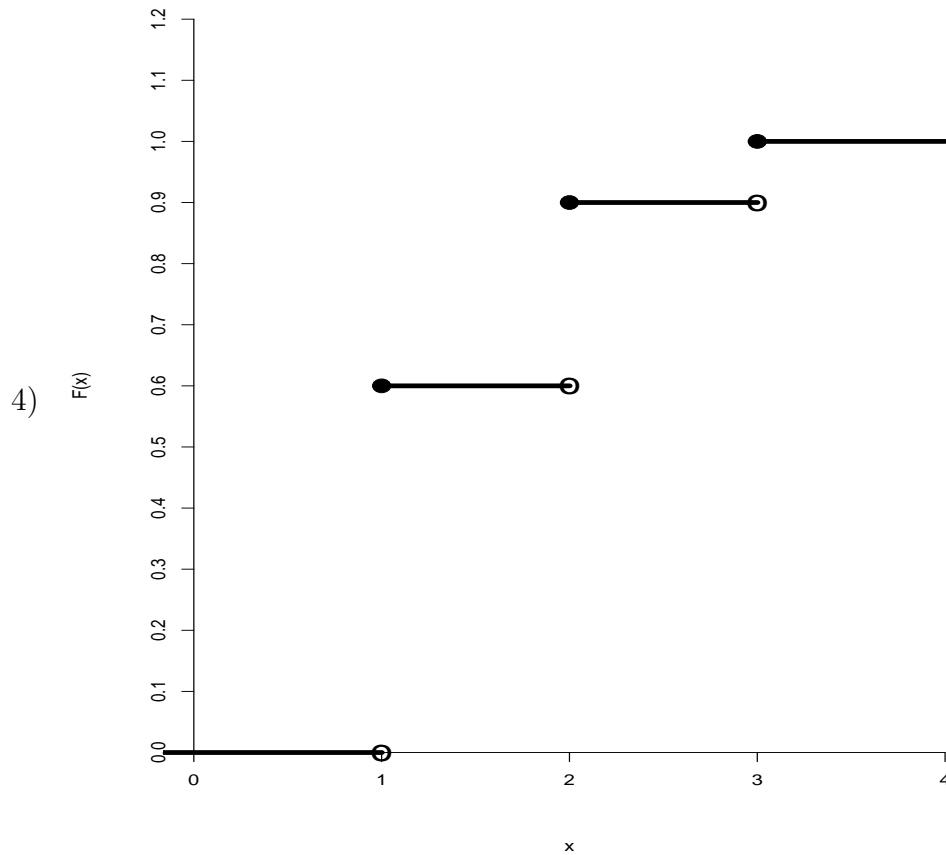
$$P(X = 3) = \frac{\binom{2}{2}}{\binom{5}{3}} = \frac{1}{10}$$

3)

$$E(X) = 1 \frac{6}{10} + 2 \frac{3}{10} + 3 \frac{1}{10} = 1.5$$

$$E(X^2) = 1^2 \frac{6}{10} + 2^2 \frac{3}{10} + 3^2 \frac{1}{10} = 2.7$$

$$\text{var}(X) = E(X^2) - E^2(X) = 2.7 - 1.5^2 = .45$$



5) Think of 5 brands as 5 balls, the best two brands as 2 black balls, and the remaining three brands as 3 white balls. The number of bottles priced at \$3 is the number of black balls when selecting 3 balls out of these 5 balls without replacement. Therefore, Y has a hypergeometric distribution with $N = 5$, $M = 2$, and $n = 3$:

$$P(Y = y) = \frac{\binom{2}{y} \binom{3}{3-y}}{\binom{5}{3}}, \quad y = 0, 1, 2$$

That is,

$$P(Y = 0) = \frac{\binom{2}{0} \binom{3}{3}}{\binom{5}{3}} = \frac{1}{10}$$

$$P(Y = 1) = \frac{\binom{2}{1} \binom{3}{2}}{\binom{5}{3}} = \frac{6}{10}$$

$$P(Y = 2) = \frac{\binom{2}{2} \binom{3}{1}}{\binom{5}{3}} = \frac{3}{10}$$

6)

$$E(Y) = n \frac{M}{N} = 3 \frac{2}{5} = 1.2$$
$$\text{var}(Y) = n \frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1} = 3 \frac{2}{5} \frac{3}{5} \frac{2}{4} = \frac{9}{25} = .36$$

7) Note that $Z = 3Y + 2(3 - Y) = 6 + Y$. Therefore,

$$P(Z = 6) = P(Y = 0) = \frac{1}{10}$$
$$P(Z = 7) = P(Y = 1) = \frac{6}{10}$$
$$P(Z = 8) = P(Y = 2) = \frac{3}{10}$$

8) Because $Z = 6 + Y$,

$$E(Z) = 6 + E(Y) = 7.2$$
$$\text{var}(Z) = \text{var}(Y) = .36$$