

## Test Problems of Calculus for Statistics 381

Note: Multivariate calculus is prerequisite for Statistics 381, which does \*not\* review or re-teach multivariate calculus, but simply makes use of it. All students taking Statistics 381 are advised to get familiar with and proficient at multivariate calculus through their independent review or study.

The following is a collection of calculus problems, which will typically be encountered in the course. Please use these problems as a test to assess your level of proficiency in multivariate calculus by yourselves.

1. 1) Find

$$\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{k-1} \quad \text{and} \quad \sum_{k=1}^{\infty} k \left(\frac{2}{3}\right)^{k-1}.$$

2) Find

$$\sum_{k=1}^{\infty} \frac{2^k}{k!} \quad \text{and} \quad \sum_{k=1}^{\infty} \frac{2^k}{(k-1)!}.$$

2. Let

$$F(x) = \begin{cases} 0, & x < 0, \\ \sqrt{x}, & 0 \leq x \leq 1, \\ 1, & x > 1. \end{cases}$$

1) Graph  $F(x)$ .

2) Find  $\lim_{x \rightarrow -\infty} F(x)$ ,  $\lim_{x \rightarrow 1^-} F(x)$ ,  $\lim_{x \rightarrow 1^+} F(x)$ , and  $\lim_{x \rightarrow \infty} F(x)$ .

3) Find  $\frac{d}{dx} F(x)$  for  $x \neq 0$  or 1.

3. Find

$$\int_2^{\infty} e^{-3x} dx \quad \text{and} \quad \int_2^{\infty} x e^{-3x} dx.$$

4. Let  $A = \{(y_1, y_2) : 0 \leq y_2 \leq y_1\}$ . Graph  $A$  and find

$$\begin{aligned} & \iint_A e^{-y_1} dy_1 dy_2, \\ & \iint_A (y_1 + y_2) e^{-y_1} dy_1 dy_2, \\ & \iint_A y_1 y_2 e^{-y_1} dy_1 dy_2. \end{aligned}$$

5. Let

$$\begin{cases} z_1 = y_1 + y_2, \\ z_2 = y_2/y_1, \end{cases} \quad y_1 > 0, y_2 > 0.$$

Express  $(y_1, y_2)$  in terms of  $(z_1, z_2)$  and find

$$\frac{\partial(z_1, z_2)}{\partial(y_1, y_2)} \quad \text{and} \quad \frac{\partial(y_1, y_2)}{\partial(z_1, z_2)}.$$